Computation of Diffusion Function Measures in q-Space Using Magnetic Resonance Hybrid Diffusion Imaging

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Background

Complex tissue organization (e.g., crossing white matter fibers) and non-mono-exponential diffusion decay (e.g., so-called fast and slow diffusion) may be described using diffusion q-space measurements to estimate the probability density function (PDF) of water diffusion. The q-space formalism was first introduced by Callaghan [1], and more recently, adaptations of this approach have been applied to diffusion imaging of the human brain on clinical scanners, i.e. diffusion spectrum imaging (DSI) [2]. This method samples a discrete range of q-space and exploits the Fourier relationship to estimate the PDF properties without any assumed model. In this study hybrid diffusion imaging (HYDI) [3], a non-uniform sampling strategy, was used to sample q-space. Measures of the PDF, including the zero displacement probability (Po), the mean squared displacement (MSD) and the orientation distribution function (ODF), were processed using new methods that are more direct and do not require the 3D Fourier Transform (3D FT).

Theory

The relationship between the diffusion PDF, $P(\vec{R})$, and the q-space diffusion signals, $E_\Delta(\vec{q})$, is a 3DFT [1], $P(\vec{R},\Delta) = FT_{3D}^{-1}[E_\Delta(\vec{q})]$, where Δ is the diffusion time. However, it is possible to estimate many of the PDF measures directly from the q-space signals without 3DFT by the central section theorem, the central ordinate theorem and scaling properties of the FT. Table 1 lists formulae for estimating PDF measures using both the conventional 3DFT and the new direct approaches. The Po is a marker of restricted diffusion [3,4]. The MSD is related to the average diffusivity in a voxel via the Einstein diffusion equation, $<\vec{R}^2> = 6\Delta\overline{D}$ [3,4]. The ODF describes the directional information of underlying biological structures [2].

Table 1. Computation of PDF measures

Measures	Conventional 3D FT approach in the displacement space	new direct computation in the q-space
Po	$Po = P(\vec{R}=0,\Delta)$ central ordinate value of PDF	$Po \approx \int_{-q_{max}}^{q_{max}} E_\Delta(\vec{q})d\vec{q}$
MSD	$MSD = \iiint P(\vec{R},\Delta)\cdot\vec{R}^2 d^3\vec{R}$ expectation value of squared displacement, $\vec{R}^2$	$MSD = \sum_{i=x,y,z}\int FT_{1D}[CQ_i]R_i^2 dR_i$ where CQ_i is the conditional function of q-space signals along i[th] (i=x,y,z) axis.
ODF	$ODF(\theta_R,\varphi_R) = \int_\rho P(\theta_R,\varphi_R,\rho)d\rho$ radial line integral of PDF	$ODF(\theta_R,\varphi_R) = MQ_{\theta_R,\varphi_R}(q_r=0).$ $MQ_{\theta R,\varphi R}$ (q_r=0) is the central ordinate value of 3D Radon transform of q-space signals along direction (θ_R, φ_R).

Materials and Methods

HYDI was performed on a healthy volunteer on a 3 Tesla GE SIGNA scanner with an 8-channel head coil and ASSET parallel imaging. The HYDI q-space scheme is described in Table 2. The diffusion-weighting (DW) pulse sequence was a spin-echo EPI sequence with cardiac gating. MR parameters: δ/Δ = 45/56 ms, Δq_r = 15.2 mm^{-1}, q_{max} = 76.0 mm^{-1}, FOV_R= 65 µm, ΔR = 6.57 µm, voxel size = 2x2 mm^2, 30 slices with slice thickness = 3 mm, TE/TR= 122/11700 ms and a total scanning time of approximately 30 min. Monte Carlo computer simulations were performed to investigate the effects of signal-to-noise-ratio (SNR) and q-space truncation on the Po and MSD measurements for both the conventional (3DFT) and the new computation methods. Six SNR levels (i.e., 10, 20, 30, 40, 50, and 100) were simulated by adding Gaussian random noise in quadrature with 100 random trials [5]. Five maximum b-values (i.e., 2800, 4375, 6300, 8575 and 11200 s/mm^2) were studied for the truncation effects. The numerical phantoms were two simple isotropic diffusions - fast diffusivity (1.15 x10^{-3} mm^2/s) and slow diffusivity (0.45 x10^{-3} mm^2/s).

Table 2. HYDI encoding scheme

HYDI	Ne	b value (s/mm^2)
Shell	1	0
1[st]	3	375
2[nd]	12	1500
3[rd]	12	3375
4[th]	24	6000
5[th]	50	9375
Total	102	

Results and Discussion

Maps of the PDF measures using the new and conventional computation methods are shown in the upper and bottom rows of Fig. 1, respectively. Both Po maps show high tissue contrast. The difference in Po maps appears to be caused by the q-space regridding method (Matlab function, griddatan) prior to FFT. The MSD map estimated using both methods have similar values in CSF and GM, whereas the WM MSD is higher using the conventional 3D FT method due to the ringing effects by truncating slow diffusing components at high q in WM. This artifact may be minimized by increasing the q-space sampling range (i.e. the maximum b-value) at the expense of scanning time. Similar results are shown in the computer simulation below. ODF profiles estimated using both conventional 3DFT and new direct methods at the splenium of the corpus callosum are in Fig. 2. The normalized ODF profiles using the new computation method appear sharper with fewer spikes and narrower waists.

The effects of SNR on the Po and MSD measurements are shown in Fig. 3. Estimates of Po with the new direct computation method showed less variance although the values were slightly overestimated for the slow diffusivity model. Q-space truncation effects are described by the plots in Fig. 4. Obviously, the slow diffusivity signal is most sensitive to q-space truncation with underestimation of Po and overestimation of MSD. The estimation biases caused by truncation are less severe for new computation methods. For the human brain study, the maximum b value was 9375 s/mm^2 (Table 2) where the MSD of slow diffusivity in WM should be more accurate using the new method. Maps of axial and radial diffusivities, estimated using inverse variance of the q-space conditional function CQ along that specific directions, are shown in Fig. 5 (a) and (b), respectively. The axial and radial directions were defined as the directions of the maximum and the minimum ODF values, respectively.

References 1. Callaghan PT Clarendon Press 1991. 2. Wedeen VJ et al. MRM 2005;54:1377-1386. 3. Wu Y-C & Alexander AL NeuroImage 2007;36:617-629. 4. Assaf Y et al. MRM 2000;5:713-722. 5. Pierpaoli C & Basser PJ MRM 1996;36:893-906.

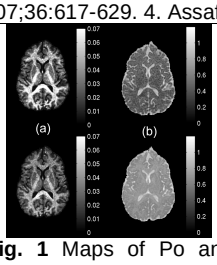

Fig. 1 Maps of Po and MSD using new direct (upper) and 3D FT (lower) method.

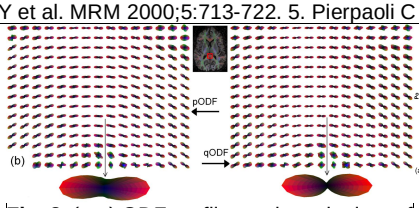

Fig. 2. (top) ODF profiles at the splenium of corpus callosum. Right: conventional radial integral of PDF. Left: new direct 3D Radon transform of q-space signals.

Fig. 3. (right) Monte Carlo noise simulation. (a) SNR vs. Po. (b) SNR vs. MSD.

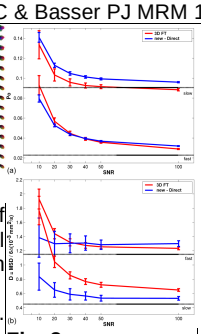

Fig. 3

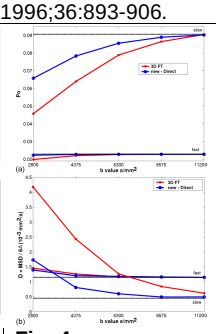

Fig. 4

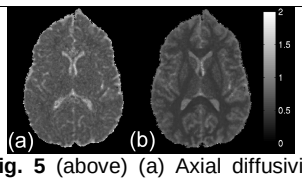

Fig. 5 (above) (a) Axial diffusivity map. (b) Radial diffusivity map. Axial and radial fiber directions were defined by the Radon transformed ODF.

Fig. 4. (left) Computer simulation of truncation effects on Po (a) and on MSD (b).