SENSE Regularization Using Bregman Iterations

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INTRODUCTION:
SENSE (1) has been widely accepted as one of the standard reconstruction algorithms for Parallel MRI. When large acceleration factors are employed, the SENSE reconstruction becomes very ill-conditioned. This problem has been addressed by Tikhonov regularization in Cartesian SENSE with some success (2, 3). However, a high-quality regularization image is needed to preserve the details, and otherwise the reconstruction is overly smooth. In this abstract, we propose a new regularization technique using Bregman iteration. Without any need for regularization images, the method iteratively refines the total variation (TV) regularization such that the regularized image has better fine scale details than using TV regularization alone (4-6). The proposed method is shown to address the over-smooth problem experienced with Tikhonov regularization and the blocky artifacts seen in TV regularization.

THEORY AND METHOD:
The proposed SENSE reconstruction with TV regularization attempts to find the best estimation of the desired image with mimimum total variation according to TV principle. Specifically, the image is reconstructed by $\vec{f}_{reg} = \arg\min_{\vec{f}} \left\{ \frac{\lambda}{2} \left\| \mathrm{E}\vec{f} - \vec{d} \right\|_2^2 + \left\| \vec{f} \right\|_{\mathrm{BV}} \right\}$ [1], where $\vec{f}$ is the entire desired image, $\vec{d}$ is the downsampled *k*-space data, $\mathbf{E}$ represents the sensitivity encoding matrix defined in (1), and λ is the regularization paramtere, with the first norm being L_2 and the second being the bounded variance norm defined as $\left\| \vec{f} \right\|_{\mathrm{BV}} = \sum_{m,n} \sqrt{\left|\nabla_x f_r^{(m,n)}\right|^2 + \left|\nabla_x f_i^{(m,n)}\right|^2 + \left|\nabla_y f_r^{(m,n)}\right|^2 + \left|\nabla_y f_i^{(m,n)}\right|^2}$ where ∇ is the gradient and the subscripts *r* and *i* denote real and imaginary parts. To solve the minimization problem in Eq. [1], we first establish the following time dependent partial difference equation $\vec{f}_t = \nabla \cdot \left(\frac{\nabla \vec{f}}{|\nabla \vec{f}|}\right) + \lambda E^H (\vec{d} - E\vec{f})$ [2], where ^{H} indicates complex conjugate transpose, and use the iterative time marching method (7) to solve Eq. [2]. The initial image determines the speed of convergence, and is chosen here to be the low resolution image obtained from the ACS data in k-space. The iterative solution leads to the TV regularized SENSE reconstruction. To further refine the image details and remove noise, we add Bregman iteration. Specifically, the above TV reconstruction procedure is regarded as the zeroth Bregman iteration, and the result is denoted by $\vec{f}_1$. In the *k*th Bregman iteration with *k*>0, the same iterative time marching method is used but to solve an updated minimization problem where the $\vec{d}$ in Eq. [1] is replaced by $\vec{d} + v_k$ where $v_k = \vec{d} + v_{k-1} - E\vec{f}_k$, with the initial conditions $v_0 = 0$, and $\vec{f}_1$ obtained from the TV reconstruction. As the Bregman iteration continues, the fine details of image are gradually reconstructed. The proposed regularization technique needs only one regularization parameter.

RESULTS AND DISCUSSION:
We tested the proposed method on a set of *in vivo* data. A 3T commercial scanner (GE Healthcare, Waukesha, WI) and 8-channel head coil (Invivo, Gainesville, FL) was used to scan a healthy volunteer with a 2D T1-weighted spin echo protocol (axial plane, TE/TR = 11/700 ms, 22cm FOV, 10 slices, 256x256 matrix). The full k-space data were acquired for reference and then downsampled to simulate a reduction factor of 4. The sensitivity profiles of the coils were estimated from the central k-space data acquired using cosine taper window (8) ($k_c = 20, \omega = 20$). The sum of the squares of the low resolution images were also used as the initial image for the iterative regularization. The reconstruction after 2 Bregman iterations and 30 fixed-point iterations within each Bregman iteration is shown in Fig. 1. For comparison, Tikhonov-regularized SENSE reconstruction (2) with a low resolution regularization image, the iterative conjugate gradient (CG) SENSE reconstruction (9) after 40 iterations, and basic SENSE reconstruction (1) are also shown. All the algorithms were implemented in MATLAB (MathWorks, Natick, WA). The results show that the proposed TV-based Bregman iterative regularization method gives the best reconstruction. In addition, the computational complexity of the proposed method is about the same as that of CG method. In our experiment, 2 Bregman iterations with 30 inner iterations took only 52 seconds on a 2.8GHz CPU/512MB RAM PC. In contrast, the running time for Tikhonov regularization, CG and basic SENSE are 22, 57 and 7.8 seconds, respectively.

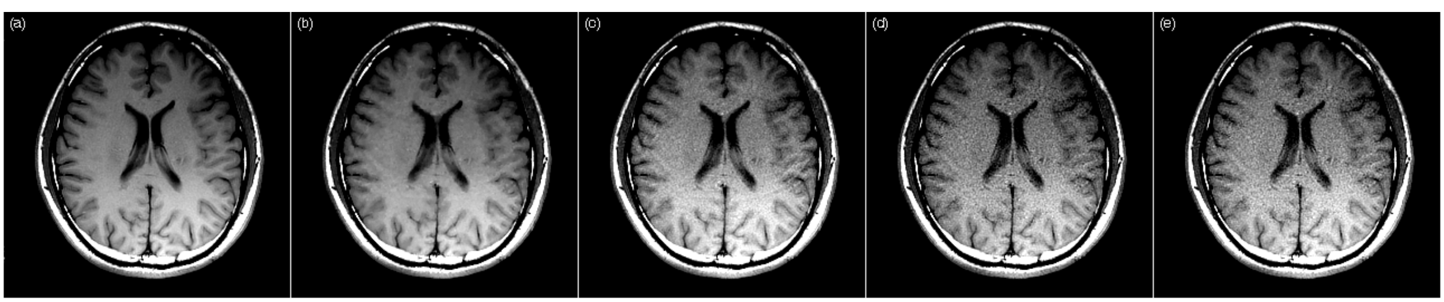

Fig. 1: SENSE recon from phantom acquired with 8 coils and a reduction factor of 4. (a) Recon from fully sampled data, (b) proposed Bregman regularized reconstruction,(c) Tikhonov regularized reconstruction with low resolution image as the regularization image (d) CG reconstruction, and (e) basic SENSE reconstruction.

CONCLUSION:
A novel regularized SENSE reconstruction method based on TV regularization and Bregman iteration is proposed to reduce the image noise amplification due to ill-condition at high reduction factors. Results show this algorithm is superior to the existing methods in terms of reconstructed image quality.

REFERENCE:
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