Influence of Regularization on Noise Amplification in Iterative SENSE Reconstruction

H. Eggers[1], and P. C. Mazurkewitz[1]
[1]Sector Medical Imaging Systems, Philips Research Europe, Hamburg, Germany

Introduction

In sensitivity encoding (SENSE) imaging, the analytical calculation of the noise amplification arising from reconstruction [1] is only practicable for Cartesian acquisitions. For other sampling patterns, a statistical estimation with Monte Carlo simulations has been proposed instead, which involves the iterative reconstruction [2] of multiple noise images. Because of the ill-conditioned nature of the underlying inverse problem, the convergence of the reconstruction is instable [4], and the obtained maps of the noise amplification depend on the number of iterations used. In this work, explicit regularization is incorporated into the reconstruction to eliminate this influence and to quantify the noise amplification in non-Cartesian imaging more objectively.

Methods

Describing reconstruction by a matrix $\mathbf{F}$ that maps *k*-space samples to image pixels, the noise covariances in image space $\mathbf{X}$ and *k*-space $\mathbf{\Psi}$ are linked by $\mathbf{X}=\mathbf{F\Psi F}^H$ [5]. The former reads $\mathbf{X}_1=\mathbf{F}_1^{-1}$ and $\mathbf{X}_2=\mathbf{F}_2^{-1}\mathbf{F}_1\mathbf{F}_2^{-1}$ for the iterative reconstruction with implicit [2,4] and explicit [6] regularization, respectively, where $\mathbf{F}_1:=(\mathbf{E}^H\mathbf{\Psi}^{-1}\mathbf{E})$ and $\mathbf{F}_2:=(\mathbf{E}^H\mathbf{\Psi}^{-1}\mathbf{E}+\alpha\mathbf{R}^{-1})$. $\mathbf{E}$ comprises the Fourier and sensitivity encoding, and $\alpha\mathbf{R}$ the explicit regularization. Although assumed in other work, $\mathbf{X}_2$ does not reduce to $\mathbf{F}_2^{-1}$. For Cartesian acquisitions, the diagonal elements of $\mathbf{X}_1$ and $\mathbf{X}_2$ serve as reference for the statistical estimation. Simulations were performed with Cartesian, radial and spiral sampling patterns following Ref. [3]. The selected results shown below were computed for a coil configuration with eight elements, placed equidistantly around the circumference of a circular field of view.

Results

The accuracy of the statistical estimation is demonstrated in Fig. 1. For both regularizations, the analytical and statistical results, presented in form of geometry factor maps [1], match satisfactorily in Cartesian imaging. The continuously increasing noise amplification encountered in the iterative reconstruction with implicit regularization is illustrated in Fig. 2. Obviously, the choice of the number of iterations has a considerable influence on the resulting noise amplification in this case. Using explicit regularization instead, this problem is effectively resolved. Geometry factor maps obtained with both regularizations are shown in Fig. 3. They exhibit a more homogeneous and lower maximum noise amplification for non-Cartesian than for Cartesian acquisitions. The most significant differences between implicit and explicit regularization are found for radial imaging. Unlike for the other sampling patterns, the noise amplification is higher with explicit regularization, which indicates a premature stop of the iteration in this case and varying effective amounts of regularization in the three cases.

Conclusions

Explicit regularization permits to attain stable convergence of the iterative SENSE reconstruction for non-Cartesian acquisitions. The reliability and comparability of the estimated maps of the noise amplification are thus improved. Previous observations on potential advantages of non-Cartesian imaging with respect to noise amplification were confirmed and substantiated using explicit regularization.

References

1. Pruessmann KP, et al. Magn Reson Med 1999; 42:952-962.
2. Pruessmann KP, et al. Magn Reson Med 2001; 46:638-651.
3. Eggers H, et al. Proc ISMRM 2005; 2429.
4. Qu P, et al. Magn Reson Med 2005; 54:1040-1045.
5. Rao CR. John Wiley & Sons 1973.
6. Eggers H, et al. Proc ISMRM 2004; 2409.

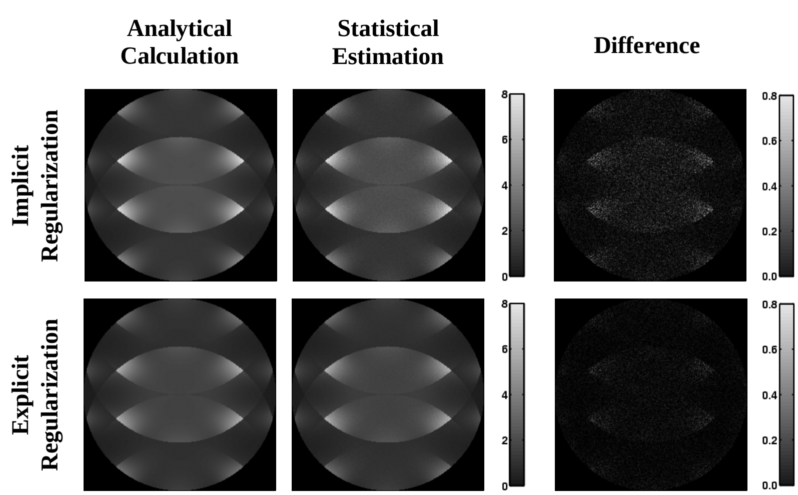

Fig. 1. Comparison of analytically calculated and statistically estimated maps of the noise amplification for a Cartesian acquisition with reduction factor 4.0.

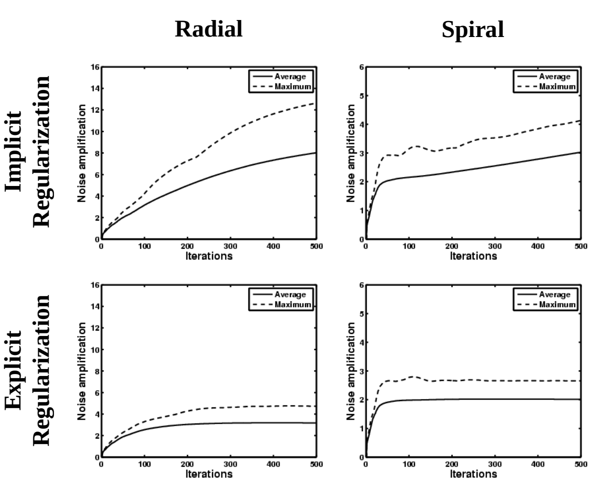

Fig. 2. Evolution of the average and the maximum noise amplification during the iterative reconstruction for radial and spiral acquisitions with reduction factor 4.0.

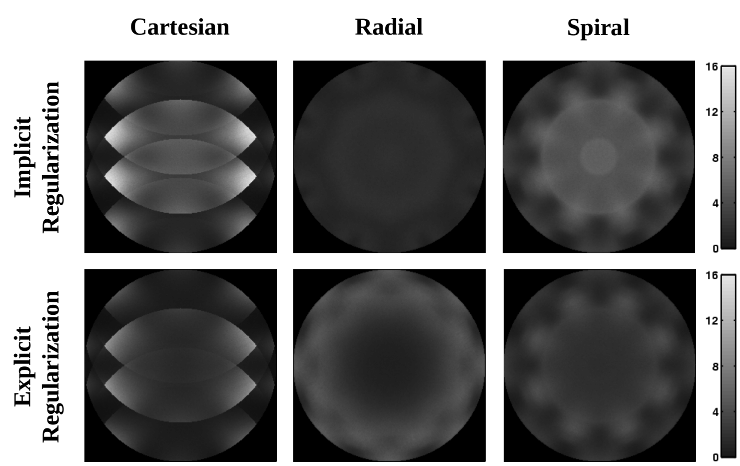

Fig. 3. Comparison of maps of the noise amplification for different sampling patterns with reduction factor 4.9.