A prospective error measure for k-t SENSE

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Introduction: Dynamic undersampled acquisition strategies [1] have shown great promise, however analysis of the errors in these methods is difficult. Generally there is a trade off between temporal and spatial artefacts, the former typically manifests as lowered effective temporal resolution and the latter as spatially distributed reconstruction errors. In k-t methods these errors are dependent on features of the object being imaged. In the case of parallel imaging techniques such as SENSE [2] the errors are well understood and can typically be quantified in terms of the "g-factor", which specifies the noise amplification directly from coil sensitivity data. Calculation of the g-factor requires only coil sensitivity maps and so can be used as a prospective tool to determine the maximum supported acceleration. No equivalent measure has been available for k-t SENSE and allied methods; this can result in ad hoc decisions about choices of imaging parameters and undersample patterns leading to variable results. We present a generalisation of the g-factor, termed g^{kt} which can help characterise performance and predict the balance of blurring resulting from temporal filtering, and noise amplification from poor coil encoding. To be used prospectively the method requires coil sensitivity maps and prior information on the full temporal bandwidth signal (usually obtained from a low spatial resolution training scan).

Theory: For SENSE with an undersample factor of R, the unfolding matrix for a group of R aliased pixels can be written as $U = (S^H \Psi^{-1} S)^{-1} S^H \Psi^{-1}$ (Eq 1), where S is the coil sensitivity matrix and Ψ is the noise covariance. The g-factor is then defined as $g_i^{\,2} = \left[(S^H \Psi^{-1} S)^{-1}\right]_{ii} \left[(S^H \Psi^{-1} S)\right]_{ii}$ (Eq 2), this is a direct measure of noise amplification caused when the coil sensitivities provide insufficient supplementary encoding. The unfolding matrix for k-t SENSE may be written $U = (S^H \Psi^{-1} S + M^{-2})^{-1} S^H \Psi^{-1}$ (Eq 3), which is identical to Eq1 except that the unfolding is now carried out in the x-f domain and the matrix M contains estimates for the signal in the aliased voxels from the un-aliased training data. We may define a new quantity for 'k-t' reconstructions that relates to Eq3 as Eq2 relates to Eq1 as $(g_i^{\,kt})^2 = \left[(S^H \Psi^{-1} S + M^{-2})^{-1}\right]_{ii} \left[(S^H \Psi^{-1} S + M^{-2})\right]_{ii}$ (Eq 4). This measure encompasses noise amplification due to bad conditioning of coil sensitivities as before, but now also includes damage that can arise from temporal filtering in the reconstruction. In the case where there is only one receiver coil (and k-t SENSE reduces to k-t BLAST) g^{kt} only describes this latter effect; in this regime it may be written $(g_i^{\,kt})^2 = 1 + \mu_i \sum_{i \neq j} \mu_j \Big/ 1 + \sum_{j=1}^{R} \mu_j$ (Eq 5) where $\mu_i \equiv \left|M_{ii}^{\;2}\right| / \psi$.

Method: To explore the properties of g^{kt} experiments have been performed using data from both numerical phantoms and in vivo images. In order to investigate the effect of undersample pattern on reconstruction error, and the ability of g^{kt} to predict in advance the best undersample pattern, time-resolved cardiac data was retrospectively undersampled using multiple sub sampling patterns and reconstructed. Numerical phantom data were used in order to investigate the spatial and temporal characteristics of g^{kt} without the confounding factor of anatomy. Reconstructions using k-t SENSE and k-t BLAST were investigated and evaluated using normalised root mean square error (E) against the ground truth.

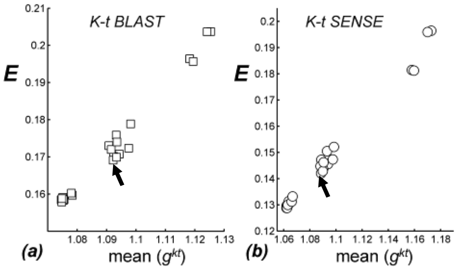

Fig 1: Normalised RMS reconstruction error versus mean(g^{kt}) for retrospectively undersampled in vivo cardiac images with R=5.

Results and Discussion: Figure 1 shows E versus the mean value of g^{kt} over the entire x-f domain for a representative cardiac data set with R=5. In this case the data were undersampled with 24 different lattices, and each point in fig 1 represents a single undersample pattern. For the k-t BLAST case the data from separate coil elements were combined prior to undersampling, and g^{kt} is calculated using the single coil version of Eq4. In both cases g^{kt} was calculated using the same low resolution training data as the reconstruction itself. There is a strong correlation between the mean of g^{kt} and the reconstruction error. The suggested undersample pattern given in ref [3] is marked with an arrow; in this case, there are patterns which lead to lower reconstruction error.

Although simple 'sheared grid' undersample lattices are common with k-t applications, it is also possible to never sample certain phase-encodes, so that some aliases are separated in space but not frequency. Since coil encoding is spatial but not temporal, this can improve reconstruction when using k-t SENSE [4]. Figure 2 illustrates this for numerical phantom data with five simulated receiver coils (placed around the object). Two different undersample patterns are illustrated, both of which have R=8 and shift four aliases to each of two frequencies. The high degree of signal overlap (top images) means that reconstruction (using k-t SENSE) relies more heavily on coil encoding than temporal filtering. Data undersampled using the pattern on the left reconstruct with relatively low errors, concentrated on the edges of dynamic features indicating temporal blurring. The pattern on the right has in addition to this a large and temporally invariant noise amplification artefact; temporal invariance can be seen from the space-time profile. The corresponding g^{kt} map at f=0 shows a similar structure; the fact that this only appears for f=0 means that from this map we could predict that the error would be temporally invariant. Errors at nonzero frequencies are more associated with blurring; the g^{kt} maps for f=1 have larger values concentrated on dynamic regions.

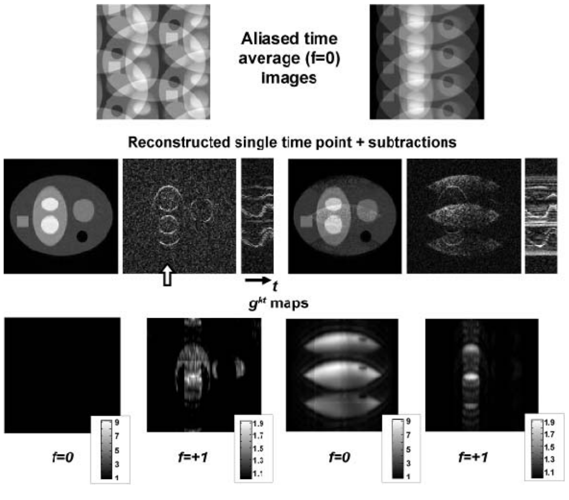

Fig 2: Numerical phantom data undersampled using R=8, such that 4 aliases are sent to each frequency. Top row: time average of undersampled images. Middle: reconstructed single time point images with subtraction from original. Bottom: maps of g^{kt} for f=0 and f=+1

Conclusions: An error metric (g^{kt}) is proposed for dynamic undersampled reconstruction, in order to predict optimal undersample patterns given the object and receiver coil setup. The strong correlation between g^{kt} and RMS reconstruction error validate this metric. Unlike other methods for optimising undersample pattern the proposed measure does not require a model [5]. Calculation of g^{kt} also allows prediction of localised error estimates or prior analysis of the nature of likely errors, enabling error reduction targeted over a fraction of the FOV or minimizing error at desired frequencies.

References: **1**. Tsao et al., MRM. 2003;50:1031-42. **2**. Pruessmann et al., MRM. 1999;42:952-62. **3**. Tsao et al., MRM. 2005;53:1372-82. **4**. Vitanes et al., Proc ISMRM(14):142. **5**. Sharif et al., Proc ISMRM(16):151.

Acknowledgement We acknowledge studentship support from the UK MRC and research grant support from Philips Medical Systems