

# Parallel Reconstruction Using Null Operations (PRUNO)

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**INTRODUCTION** As an auto-calibrated k-space parallel imaging method, GRAPPA (1) has shown its advantages in some applications when accurate coil sensitivity maps are difficult to obtain. However, there are two main drawbacks for GRAPPA, especially under large imaging acceleration. First, accurate data calibration requires many ACS (Auto-Calibration Signal) lines, which essentially lowers the actual reduction factor. Second, at high reduction rates, missing data are synthesized using data acquired at a far distance which have low correlation with the missing data. The low correlation is a result of the intrinsic narrow-banded sensitivity profiles (3). To overcome these shortcomings, we propose an iterative k-space-based Parallel Reconstruction Using Null Operations (PRUNO). In PRUNO, some local null operators are applied on all k-space locations within a neighborhood rather than only acquired lines. By using these null operators, the reconstruction problem is formulated as estimating missing data from available k-space data by solving a system of linear equations. We also demonstrate that it can be solved efficiently and accurately with a conjugate gradient method. Our preliminary simulation and *in vivo* results suggest that PRUNO can significantly improve the accuracy of image reconstruction compared to GRAPPA.

**METHODS** In parallel imaging, the coil sensitivity profiles are usually very smooth and k-space spectra are very compact. As a result, nearby k-space samples from multi-coils are highly correlated and some subsets of them may be approximately linearly dependent. In other words, there exists a non-zero, linear and shift-invariant operator, which nulls a selected local set of samples. By choosing different neighbor templates, we can obtain multiple null operators. The coefficients of each operator can be calibrated on ACS similar to GRAPPA. These operators essentially reveal the correlations among all samples and we can use them to formulate a linear equation  $Nx = 0$ . [1]

Here  $N$  is a sparse encoding matrix which concatenates all null operators. And  $x$  is the vectorized desired full-grid multi-coil k-space data. By splitting acquired samples and missing samples into two categories, we can reorder  $x$  and  $N$  accordingly.

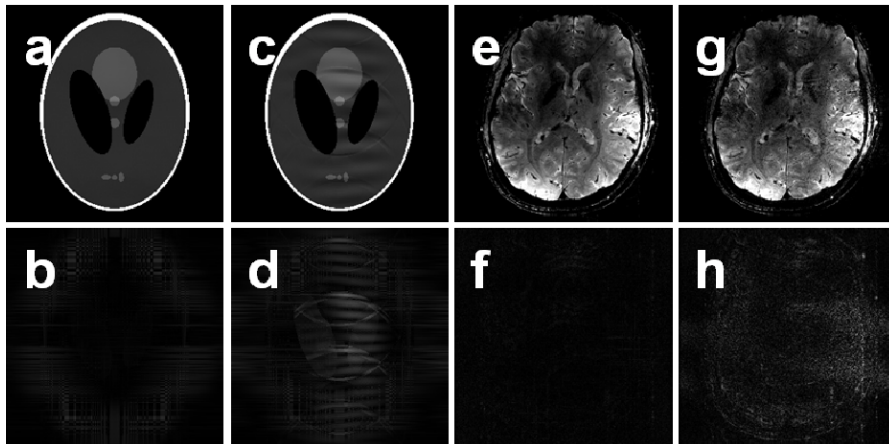
And the equation can be decomposed as 
$$\begin{bmatrix} N_m & N_a \end{bmatrix} \begin{bmatrix} x_m \\ x_a \end{bmatrix} = 0. \quad [2]$$

Here the two subscripts  $m$  and  $a$  represent missing and acquired, respectively. Obviously, the goal of the reconstruction is to solve  $x_m$ , and the final linear equation turns to be  $(N_m^* N_m) x_m = -N_m^* N_a x_a$ . [3]

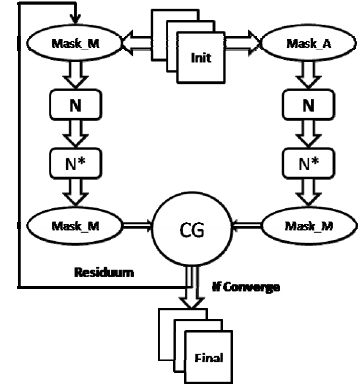
To guarantee that the system is overdetermined, we would need at least  $\lceil (\text{number of coils}) * (R-1)/R \rceil$  null operators. There are many options on how to choose a set of neighbor templates. One good choice is to simply use a set of GRAPPA operators (2) by setting the coefficient of each target sample as -1. Since both  $N_m$  and  $N_a$  are sparse, this equation can be solved using direct matrix inversion if neither the image size nor the number of coils is extremely large. It can also be solved efficiently by using a conjugate gradient method because each null operation is simply composed of convolutions and additions, which is similar to a non-Cartesian SENSE reconstruction (4). An iterative reconstruction scheme is shown in Fig 1.

**RESULTS** Our PRUNO method has been applied to both simulation and *in vivo* data. We used a matrix size of 256 and 8 channels for both experiments. The simulated data were generated using pre-measured coil-sensitivity profiles. The *in vivo* scan was performed by using a spoiled gradient recalled echo sequence on a GE 7T scanner. To verify the performance of our algorithm, full k-space samples were first acquired for each case and the data were subsampled afterwards. 3 ACS lines were used at a reduction factor of 4 for both experiments. Fig 2 shows the reconstruction results of both GRAPPA and PRUNO. We can see that under this high acceleration, GRAPPA either cannot completely unfold the aliasing or produce some signal cancellation on the final sum-of-squares images, whereas PRUNO reconstructs images without visual artifacts.

**DISCUSSION AND CONCLUSION** We have demonstrated a GRAPPA based Cartesian parallel imaging method, which shows an improvement to the image quality especially at high acceleration rates. We also introduced a fast iterative algorithm to solve the PRUNO reconstruction. As illustrated by Eq [2], PRUNO gives a more generalized optimal solution than the conventional GRAPPA. Due to the smoothness of coil sensitivity profiles, only "small" null operators are usually necessary in PRUNO. More intuitively, small null operator means missing data is always synthesized from adjacent neighbors. Another advantage of PRUNO is that it doesn't require a large number of ACS lines and the number of necessary ACS lines doesn't depend on the reduction factor. According to our results, 2 to 4 ACS lines are sufficient for most reconstructions even with large reduction factors.



**Figure 2:** Comparison of PRUNO and GRAPPA. The first row shows the sum-of-squares reconstruction results using PRUNO and GRAPPA at 4-fold acceleration. Each image is compared with its corresponding Fourier reconstruction from full samples. The resulted magnified error images are displayed in the second row (shown at 5X). Among these images, the first and third columns are reconstructed using PRUNO. And the other two columns are results from GRAPPA.



**Figure 1:** The iterative algorithm for PRUNO reconstruction. Here  $N$  and  $N^*$  represent null operation and its conjugate respectively.  $Mask\_M$  and  $Mask\_A$  refer to masking out only missing samples or only acquired samples respectively.

## REFERENCES

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