# karhunen-Loeve transform filter to improve signal to noise ratio in dynamic cardiac imaging

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**Introduction:** Fast imaging in MRI is usually achieved at the expense of SNR. Noise filters that help improve image SNR (e.g., low pass filter in spatial or temporal frequency domains) are usually associated with edge blurring. Statistical filters such as the median filter are effective at removing salt and pepper noise with little reduction in image sharpness. We propose a new filter for dynamic images based on the Karhunen-Loeve transform (KLT) (principle component analysis) along the temporal dimension [1] to remove random noise without compromising the information content of the image. The filter does not require a priori information or training data [2]. Numerical simulations were used to determine the optimal eigenimage cutoff. The denoising effect was tested in one series of real time cardiac cine images.

**Theory:** Image decomposition Given a series of dynamic images $\boldsymbol{I}_j$, with j = 1, …, $c$, and size of each image $m$ x $n$, a matrix $\boldsymbol{A}$ (size $c$ x $mn$) can be constructed whereby each image in the series becomes one row of matrix $\boldsymbol{A}$. Let $\boldsymbol{B} = \boldsymbol{A}\boldsymbol{A}^T$. $\boldsymbol{B}$ is a $c$ x $c$ positive-semidefinite matrix. Its eigenvalue problem is:

$$\boldsymbol{BE} = \boldsymbol{\Sigma E}. \qquad (1)$$

Here, $\boldsymbol{\Sigma}$ is a diagonal matrix containing the eigenvalues $\lambda_i$ of $\boldsymbol{B}$. $\boldsymbol{E}$ is a unitary matrix, and each column of $\boldsymbol{E}$ is an eigenvector. One eigenvector can reconstruct one eigenimage $\boldsymbol{J}$ as:

$$J_i = \sum_{j=1}^{c} E_{ji} I_j \qquad (2)$$

Each image in the dynamic image series is a linear combination of all the eigenimages.

Image filtering The eigenimages in Eq.(2) are orthogonal to each other. They are analogous to the fundamental and harmonics of a signal spectrum. The eigenvalues ($\geq 0$) therefore weight the relative importance of each eigenimage to the series of dynamic images. In dynamic imaging, (quasi-periodic) structures appear throughout the series but noise is random. The first few eigenimages contains most of the signal while noise level is uniform in all eigenimages (see Fig. 1a). The i$^{th}$ filtered image in the dynamic series is therefore given by keeping only the eigenimages associated to the largest $d$ (<c) eigenvalues:

$$I_i = \sum_{j=1}^{d} E_{ij} J_j \qquad (3)$$

Note that $\boldsymbol{E}^T = \boldsymbol{E}^{-1}$(assuming $\boldsymbol{B}$ and $\boldsymbol{E}$ have full rank for simplicity).

The eigenimages associated with small eigenvalues are discarded. The noise level in filtered images will be lower if they are reconstructed from fewer eigenimages.

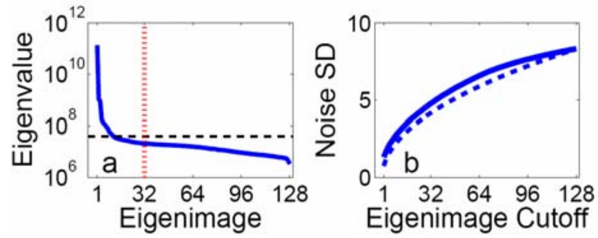


Fig. 1 **a.** Sorted eigenvalue associated with all eigenimages. Most of the energy is concentrated in the first few eigenimages. Horizontal dashed line represents the noise variance; vertical dotted line is the optimal cutoff chosen. **b**. The averaged noise standard deviation (SD) in the simulated images as a function of eigenimage filter cutoff. The dashed line represents numerical simulation images corrupted by white noise only; the solid line represents real time cardiac cine images.

**Methods:** Simulation A series of 128 images were generated simulating periodic cardiac motion. The images were corrupted by white noise with the same standard deviation (SD). The KLT filter was applied over the entire range of possible eigenimage cutoffs and noise SD measured.

Application to Cine Imaging

The experimental study was done on a 1.5T MRI system (Avanto, Siemens, Germany). A 128 image cine series was acquired using real time TSENSE SSFP with SENSE acceleration factor 5 and without ECG gating. The imaging parameters were: voxel size = 1.48 x 1.48 x 8 mm$^3$ (256 x 208 matrix, 8.0mm thick slice), flip angle = 70 degrees, temporal resolution = 65.5 ms, TE = 1.14 ms, pixel bandwidth= 1300 Hz. The original cine images were filtered with a cutoff of 32 eigenimages (25%). The SNR was evaluated by calculating SD and mean grayscale ratio of blood pool. The differences between the original and the filtered images were checked for the appearance of any coherent structures (Fig. 2). To investigate if the edge sharpness were preserved, we also demonstrate grayscale profile vs. time images.

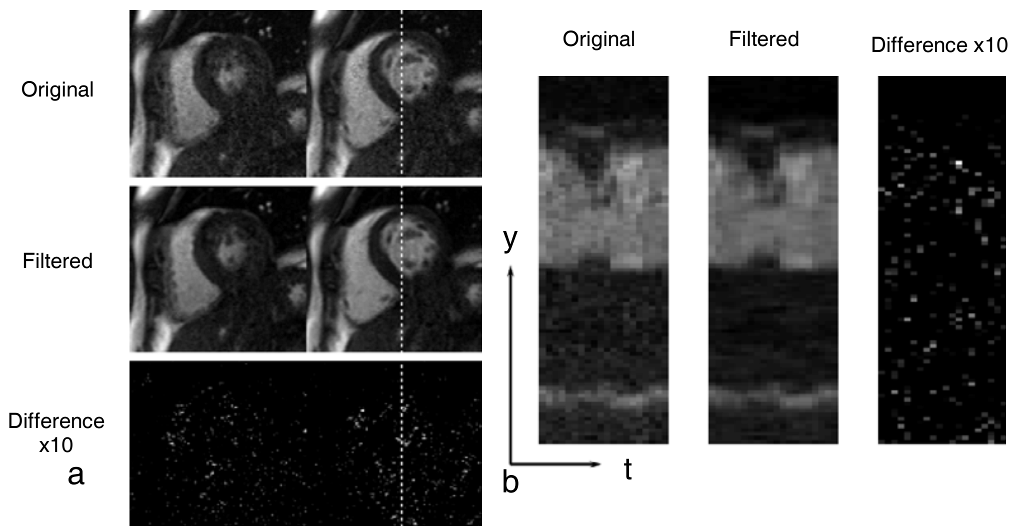


Fig. 2a The original images (top), filtered images (middle), and the differences (bottom) of two frames from a cardiac cine. b. The corresponding spatio-temporal plots of the white line marked in 2a.

**Results:** The noise SD in numerical simulation is proportional to square root of the eigenimage number cutoff (Fig. 1b). The noise reduction effect in real time cardiac cine is close to numerical simulation. Close to a 40% SNR increase was achieved in real time cardiac cine without any visible artifact (Fig. 2a). The differences between the original and filtered images are random. No edge blurring appears (Fig. 2b). If less eigenimages are used in the filtering, a higher SNR can be achieved with some blurring of moving edges. No matter how aggressive the filtering is, all the features of stationary tissue remain sharp.

**Discussion:** We propose a new post-processing filter based on KLT that can increase SNR in dynamic images without introducing image distortion. From our experiments, to achieve around 40% SNR increase, the eigenimages cutoff should be in the range of 50% to 25% of total. This method is a "smart averaging" method with no need for registration or a priori information. However, since the filter exploits the temporal relationship of the images, the dynamic images should be slowly changing or quasi-periodic. KLT filtering of dynamic images may permit recovery of SNR lost by acquisition acceleration methods, without addition of artifacts or significant loss of spatial or temporal information..

**References:** [1] Narayanan, M.V., et al., IEEE Trans. Nuc. Sci., 1999:46:1001-1008. [2] Tsao, J. et al. Proc. ISMRM. 14th Annual Meeting, 2006.