Quantization noise in MRI acquisition

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INTRODUCTION The theory and modeling of analog to digital quantization is well studied and understood. In most imaging applications, quantization happens in the spatial domain, where the quantization error can be generally assumed to be uniformly distributed random variables in the range $[-\Delta/2,\Delta/2]$, where Δ is the quantization step, uncorrelated with each other and with the input signal. The assumptions for this are: large number of quantization levels, small quantization step, and a smooth input probability density function (1). If these conditions are not satisfied, they can be forced by adding a dither signal (2) that produces images perceptually more appealing (3). In contrast, in MRI there are significant differences that could make the above conditions no longer valid. First, MRI acquisition and quantization occur in the frequency domain. As a consequence, in the image domain the quantization noise is not white anymore, because the quantization error exhibits a dependency of position in k-space. Second, the dynamic range in k-space is usually large. For frequency positions far away from the origin, the quantization step can be comparable or even larger than the variability of the data. This is especially true for 3D acquisitions. In this work we provide a theoretical background to study the quantization noise under these conditions and show one possible way to improve the perception of images affected by non uniform quantization noise. It is worth mentioning that quantization noise is not generally a critical issue in 2D MRI acquisition where the quantization step is relatively small. However, it is more relevant in the case of 3D acquisition or when low cost hardware is the only alternative and higher quantization intervals must be accepted.

BACKGROUND In k-space, quantization noise does not distribute uniformly across all the frequency ranges. In the low frequency region, where high spectral peaks can be found, quantization noise follows a uniform distribution, but in high frequency regions the noise is dominated by the statistical information of the signal, and not by the quantization process. Figure 1 shows the quantization noise derived by subtracting the original k-space from the quantized raw data in 10 bits. A direct consequence of this is that common techniques used to cope with the quantization noise, such as dithering, can not be applied to the frequency domain in the same manner that it is traditionally used in the spatial domain. Let M be the Fourier transform of our object of interest and Q the quantization process. The quantization noise is Nq, a uniform distribution in $[-\Delta/2,\Delta/2]$. Let M' = Q(M) be a quantized version of M. Let $\sigma(k)$ be the standard deviation of the k-space signal. Then, if $\Delta < \sigma(k)$ for all k (case 1), we have that M' = M + Nq. Taking the Fourier transform we obtain m' = m + F(Nq), where F(Nq) is the Fourier transform of the quantization noise. Because Nq is white and uniformly distributed, we can assume F(Nq) to be Gaussian white noise, denoted by Ng. This corresponds to the typical case of quantization, where all conditions for additive white quantization noise are satisfied. In this case, the quantized object is simply the original object plus Gaussian white noise, which is fairly benign. However, if $\Delta > \sigma(k)$ for some k (case 2), we have a different situation. Here we have M' = M + Nq for low k's and M' = 0 for high k's, as it is shown in Figure 1. If we consider the quantization error we observe M – M' = -Nq for low frequencies and M – M' = M for high frequencies. Taking the Fourier transform we obtain m' = m + LP{Ng} - HP{m} = LP{m} + LP{Ng}, where LP{ } denotes a low pass filter operation and HP{} a high pass. Thus, the quantized object in the image domain can be considered as the sum of a low pass version of a Gaussian white noise signal plus a low pass version of the object. In other words, the low frequency region of the object is as in case 1. The final appearance of the image for case 2 can be perceptually unpleasant because the cut-off frequency of the low pass filter depends on the original object spectrum, therefore the quantization noise is correlated to the object which makes it an artifact. Even though it means to add more noise we propose to add a complementary high pass noise in order to make the total quantization noise (quantization error + added noise) as white as possible. We then have m'' = m' + HP{Ng}.

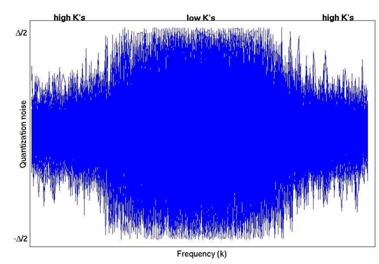

Figure 1: Quantization noise in k-space

METHOD We added uniform noise to the acquired k-space after the quantization process, only in regions of the k-space where the signal statistics are predominant (values lower than Δ). Given a quantized k-space M', the goal was to generate a modified k-space M'' whose quantization noise would follow a uniform distribution for all frequencies. In order to add the proper noise we estimate thresholds Tx, Ty and Tz where the quantization error shifts from having a uniform distribution to being dominated by the signal. These thresholds vary depending on the object being analyzed and can be estimated by inspection from M'.

Figure 2: (a) Original slice showing zoom area **(b) Zoom: original image** **(c) Zoom: image reconstructed from quantized raw data** **d) Zoom: same as (c) plus the proposed noise**

RESULTS We tried our method with a 192x192x96 3D image of a heart. We quantized the k-space using 10 bits. 99.38% of all voxels of the k-space fell below the first quantization step. Figure 2a shows a slice of the 3D data taken at the middle of the Z coordinate. Figures 2b, 2c and 2d show an amplified region of the original image under different conditions (shown inside a square in 2a). Figure 2b corresponds to the original image, figure 2c to the quantized image and figure 2d to the image obtained from the modified M''. The mutual information between the quantized and original images corresponds to a 37.5% of the maximum achievable and the mutual information between the modified and the original is only 29.5% of the maximum. However, 2d is perceptually preferred over 2c. This is due to the fact that the quantization noise present all over the image in the case of 2c contains structural information about the original image, while in 2d, the noise is simply Gaussian and not depending on the object.

CONCLUSIONS We have conducted an analysis of the effects of the quantization noise in k-space and provided a way to improve the perception of the distorted image due to this non uniform quantization. The improvements are not significant from a mathematical information point of view, but they are perceptually relevant. More detailed models of the quantization error in MRI acquisition could improve the perception of MRI even more.

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