Accurate Noise Bias Correction Applied to Individual Pixels

A. Cardenas-Blanco[1], M. Nezamzadeh[2], C. Foottit[2], and I. Cameron[2,3]

[1]Ottawa Health Research Institute, Ottawa, Ontario, Canada, [2]Physics, Carleton University, Ottawa, Ontario, Canada, [3]Diagnostic Imaging, The Ottawa Hospital, Ottawa, Ontario, Canada

Introduction

It is well known that magnitude Magnetic Resonance (MR) data are Rician distributed[1]. When the Signal-to-Noise Ratio (SNR) is low the probability density function (PDF) for these data behaves like a Rayleigh function. The corresponding mean, <M>, will be an overestimation of the true value, A, and hence if used for the calculation of quantitative MR parameters, such as the Apparent Diffusion Coefficient (ADC) or the spin-spin relaxation time, T_2, will lead to inaccurate results. In the past, several different approaches for the reduction of noise effects in MR images have been proposed[1,2,3]. This abstract presents a new noise bias correction technique that outperforms those presented previously and improves the accuracy of computed parameters such as ADC and T_2 by reducing the effects of the rectified noise bias (RNB) on the MR signal. It also has the important advantage that it can be used to accurately correct individual pixel values and not just region of interest (ROI) values.

Theory

The MR signal on the real and imaginary channels is the sum of the true signal and zero mean Gaussian noise. Since the Fourier Transform (FT) is linear, the FT operation does not change the PDF characteristics of the real and imaginary data, (i.e., both remain Gaussian). However, the calculation of pixel magnitudes rectifies the noise and transforms the distribution of data points such that the PDF becomes Rician rather than Gaussian[1]. This change also causes the mean of the PDF to be shifted from the true value, A, by an amount that we shall call the rectified noise bias (RNB). An expression A can be derived[3] (see Eq. 3) by combining the expression for $\langle M^2\rangle$, the second moment of the Rician distribution (Eq. 1), and the variance of the measured pixel magnitudes, σ^2_r, as a function of the correction factor[3], $\xi(\theta)$, and the variance of the Gaussian noise, σ^2_g (Eq. 2). This equation is exact and can be used to correct the measured signal magnitudes for RNB as long as the value of <M> and σ_g are known. This can be realized in practice if relatively large ROI's are used but with smaller ROI's, which provide poorer estimates of <M>, imaginary values for A will frequently occur. It is completely impractical to use Eq. [3] for single pixels when the SNR is small. This problem can be avoided by using the binomial expansion of the square root (Eq. 5) as proposed by Nezamzadeh *et al*[2]. Equation [5] is mathematically equivalent to Eq. [3] and, given accurate values of σ_g and <M>, yields exact values for A (see Table 1).

$$\langle M^2\rangle = A^2 + 2\sigma_g^2 \quad [1]$$

$$\sigma_r^2 = \langle M^2\rangle - \langle M\rangle^2 = \xi(\theta)\sigma_g^2 \quad [2]$$

$$A^2 = \langle M\rangle^2 - q^2\sigma_g^2 \quad [3]$$

$$q^2 = (2-\xi(\theta)) \quad [4]$$

Eq. [5] can be interpreted as a linear correction applied to the mean of the PDF. If we make the substitution $\langle M\rangle \rightarrow M_j$ in the first term, where M_j is an individual pixel value, the resulting equation (Eq. 6) gives a linear correction that can be applied to each pixel value independently. Note that the correction is the same as for the mean of the PDF in Eq. [5]. This causes all pixel values to be shifted by the same amount. Thus, the shape and the variance of the distribution of corrected data, $\tilde{A}_{NCC}$, are the same as for the uncorrected data, M_j, however, the mean of the corrected data is now equal to A – i.e. the RNB has been removed. With this expression, an ROI can be used to calculate the correction accurately but the correction can then be applied to individual pixels. To achieve the maximum precision in the binomial expansion, a convergence *criterion* was implemented to automatically determine the number of terms to be used.

$$A = \langle M\rangle - \langle M\rangle\left[\frac{1}{2}\left(\frac{q\sigma_g}{\langle M\rangle}\right)^2 + \frac{1}{8}\left(\frac{q\sigma_g}{\langle M\rangle}\right)^4 + \ldots\right] \quad \text{for } \frac{\langle M\rangle}{q\sigma_g} > 1 \quad [5]$$

$$\tilde{A}_{NCC,j} = M_j - \langle M\rangle\left[\frac{1}{2}\left(\frac{q\sigma_g}{\langle M\rangle}\right)^2 + \frac{1}{8}\left(\frac{q\sigma_g}{\langle M\rangle}\right)^4 + \ldots\right] \quad \text{for } \frac{\langle M\rangle}{q\sigma_g} > 1 \quad [6]$$

Results and Conclusions

To test the effectiveness of the correction scheme 500,000 element real and imaginary vectors were created with a standard deviation of $\sigma_g = 1$. From these the magnitude vectors were computed. For low values of SNR (0.0 to 0.5), it is clear that <M> > A (see Table 1). Table 1 also compares estimates of A computed using the correction scheme proposed here ($\tilde{A}_{NCC}$) with those of Gudbjartsson and Patz[2] ($\tilde{A}_{GP}$) and Koay and Basser[3] ($\tilde{A}_{KB}$) for the case where accurate values of <M> and σ_g are used. The $\tilde{A}_{NCC}$ and $\tilde{A}_{KB}$ values are in good agreement and are very close to the true value, A, while the $\tilde{A}_{GP}$ values are less accurate. The number of terms required to reach convergence is shown in parentheses in Table 1 for each SNR considered.

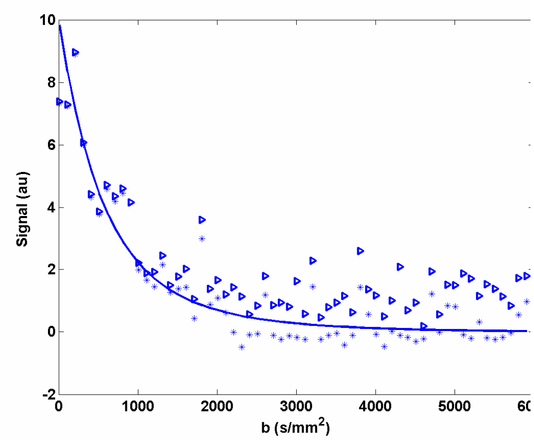
In order to determine whether the new correction scheme improves the accuracy of diffusion parameters computed from MR image data, Eq. [6] was used to correct a simulated bi-exponential diffusion decay. Figure 1 shows the uncorrected (triangles), corrected (stars), theoretical (solid line) and fitted decay data (dashed line) plotted using both a semi-log scale (blue) and a linear scale (green). Each data point represents a single pixel value. The value of <M> used to compute the correction term in Eq. [6] was calculated for ROI's ranging from 9 to 225 pixels. The fit parameters obtained using a Levenberg-Marquardt algorithm to fit the corrected and uncorrected decay data are given in Table 2. The corrected curve in Figure 1 corresponds to the 9 pixel case. From Figure 1, it can be appreciated that the corrected data (stars) are in better agreement with the theoretical curve (solid line) than the uncorrected data. For this example, we were unable to obtain a reliable fit to the uncorrected data - since D2 is negative neither fit parameter value is reliable. From these results it can clearly be seen that this new correction scheme effectively reduces rectified noise bias and increases the accuracy of diffusion parameters calculated from the MR data – *even when the correction is applied to individual pixels*.

Table 1

A	<M>	$\tilde{A}_{GP}$	$\tilde{A}_{KB}$	$\tilde{A}_{NCC}$ (iter)
0	1.253	1.035	0	0.007 (170)
0.1	1.256	1.038	0.088	0.070 (170)
0.2	1.265	1.046	0.193	0.183 (81)
0.3	1.281	1.059	0.295	0.289 (48)
0.4	1.302	1.078	0.396	0.391 (32)
0.5	1.33	1.102	0.497	0.493 (24)

Table 2

	Theoretical	Uncorrected	9 Pixels
D_1(mm^2/s)	2.00E-03	1.90E-03	(2.0)E-03
D_2(mm^2/s)	0.80E-03	fail	(0.80)E-03



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