Quadrature TV+L_1 Regularization for MR Image Denoising

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Introduction

The presence of noise in MR images unavoidably diminishes their potential for both manual and automated assessment of pathology. While a sizeable number of image denoising techniques have been developed or modified specifically for MRI, most of these existing methods are of minimal diagnostic value as they tend to degrade structure edges and can leave residual artifacts in lieu of the removed noise. Recently, it has been shown that total variation-regularized L_1 function approximation (TV+L_1) not only provides excellent noise removal with minimal residual but that contrast and morphological preservation of image structures can be analytically guaranteed for certain object classes [1]. We propose application of the TV+L_1 denoising method in quadrature to clinical MR images and discuss optimal blind parameterization of this model for fully-automated implementation. Examples are given highlighting the superiority of this new technique over well-established methods such as adaptive filtering, wavelet denoising, and anisotropic diffusion, and its potential for application in clinical practice.

Methods

Suppose f is an image that has been corrupted by an Additive White Gaussian Noise (AWGN) field, $n \sim N(0,\sigma^2)$. The TV+L_1 method for image denoising consists of solving the following convex optimization problem:

$$u = \arg\min_u \left\{ \int_\Omega |\nabla u| dx + \lambda \int_\Omega |u - f| dx \right\} \qquad (1)$$

where u is the recovered image and λ is a non-negative regularization parameter that controls the tradeoff between solution smoothness and fidelity to the measured image. Following standard techniques from the calculus of variations, a solution to (1) is given by the Euler-Lagrange equation,

$$\nabla \cdot \frac{\nabla u}{|\nabla u|} - \lambda \operatorname{sgn}(u - f) = 0 \qquad (2)$$

subject to homogeneous Neumann boundary conditions. Although ideal parameterization of λ typically requires *a priori* knowledge of σ^2, it can be shown that the associated Minimum Mean Square Error (MMSE) estimator of the value satisfying the Karush-Kuhn-Tucker (KKT) sufficient conditions for a global minimum is given by

$$\tilde{\lambda} = \frac{1}{|\Omega|} \int_\Omega \left| \nabla \cdot \frac{\nabla u}{|\nabla u|} \right| dx \qquad (3)$$

which, unlike analogous estimators for the TV+L_2 model, is completely independent of σ. Consequentially, an excellent approximation of the ideal u in (2) can be achieved blindly and within a fully-automated framework.

Given a complex MR image and exploiting the fact that noise on each quadrature channel consists of an AWGN random field, the real and imaginary image channels can be denoised independently by solving (2) and utilizing the identity in (3). Unlike magnitude-specific methods such as [2], the quadrature approach [3] can naturally be used for phase-based applications such as MR elastography. To numerically compute the solution of (2), we employed a simple artificial time marching scheme [1] subject to standard Courant-Friedrichs-Lévy (CFL) timestep bounds; iteration was automatically terminated according to the Morozov discrepancy principle.

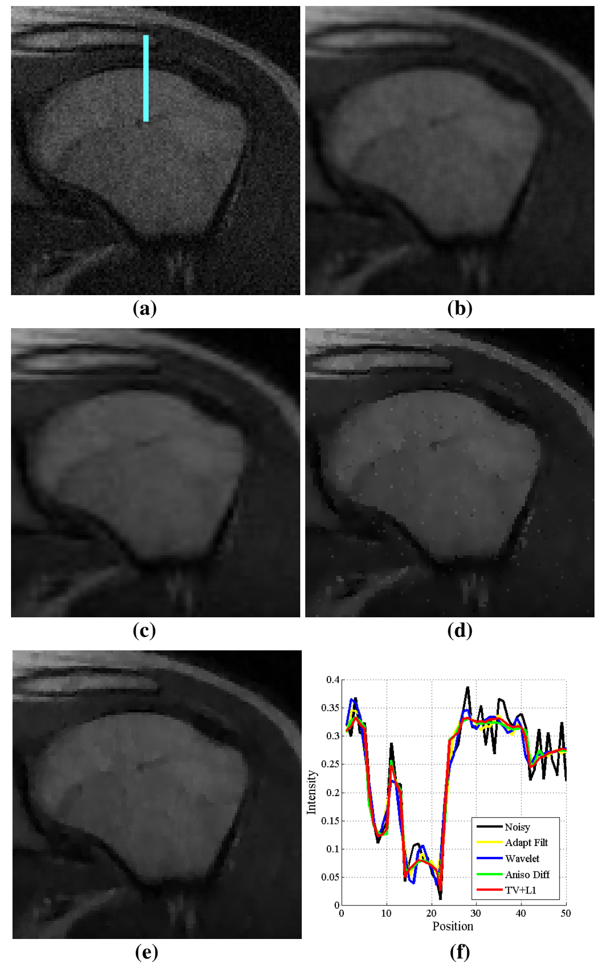

Fig 1. Noisy image of the humeral head (a) and denoising by adaptive filtering (b), wavelets (c), anisotropic diffusion (d), and TV+L_1 regularization (e). The profile in (f) was taken along the cyan line in (a).

Results

A comparison was made between the presented TV+L_1 method and adaptive Wiener filtering [4], BayesShrink wavelet denoising [5], and anisotropic diffusion [6] for a number of 2D MR images covering different anatomical areas, with all methods being applied in quadrature. The total variation (TV) of the denoised images was measured to assess solution smoothness and the mutual information (MI) between the denoised image and the residual was calculated to determine if significant structural information had been removed during the denoising process. In all cases, both the TV and MI were the lowest for the TV+L_1 method, asserting that this approach can achieve the greatest degree of noise removal while best preserving image morphology. Additionally, unlike wavelet and anisotropic diffusion techniques, the TV+L_1 method leaves little to no artifact from the denoising process. Figure 1 shows an example result for denoising of an image of the humeral head, and table of the TV and MI measures for the TV+L_1 and compared methods can be seen in Table 1.

	Adaptive Wiener Filter	Wavelet Denoising	Anisotropic Diffusion	TV+L_1 Denoising
TV	1.5191	1.5375	1.3825	1.3428
MI	0.1188	0.1266	0.1178	0.1085

Table 1. Numerical results for example images shown in Figure 1.

Summary

We have presented a novel method for noise removal in MRI that not only provides superior results to existing methods but can also be applied with no *a priori* knowledge of noise statistics. Additionally, the contrast and morphological preservation characteristics of this model naturally extend the range of this method to specific applications such as MR angiography.

References

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