Improved k-t BLAST using FOCUSS

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Introduction : Recently, there have been growing interests in applying the compressed sensing theory to MR imaging applications. The compressed sensing theory tells us that accurate reconstruction is possible even from the samples dramatically smaller than Nyquist sampling limit as long as the unknown image is sparse or compressible. The main contribution of this paper is to show that a sparse reconstruction method called the FOCal Under-determined System Solver (FOCUSS) is very effective for dynamic imaging from compressed sensing perspective since the dynamic images such as cardiac motions are usually sparse in spectral domain. The new algorithm called k-t FOCUSS is developed for dynamic MR imaging which is asymptotically optimal from compressed sensing perspective. Also, this algorithm explains how k-t BLAST works. Actually, k-t BLAST has been recognized for its performance. But there was no enough explanation why the algorithm works. The simulation results show that k-t FOCUSS is a generalization of k-t BLAST and outperforms that.

Theory : FOCUSS is an algorithm designed to obtain the sparse solutions to the underdetermined linear inverse problem

$$v = F\rho \tag{1}$$

In dynamic MR, v corresponds to measurement on (k-t) space and ρ means unknown sparse support on (x-f) space. Therefore, F implies 2-D Fourier transform. Now, let us consider the following optimization problem: Find $\rho = Wq$ (2)

Where W is a weighting matrix, and q is a solution of the following constrained minimization problem:

$$\min \|q\|_2 \text{ , subject to } \|v - FWq\|_2 < \varepsilon \tag{3}$$

To solve above problem, initial W should be estimated first. To get initial low resolution estimate, we employed the random sampling pattern with more samples around low frequency region and applied zero-padded 2-D Fourier transform.

The constrained optimization problem can be converted into the un-constrained optimization problem using Lagrangian multiplier, providing a cost function

$$C(q) = \| v - FWq \|_2^2 + \lambda \| q \|_2^2 \tag{4}$$

In a slightly different formulation, ρ is initialized with non-zero values $\overline{\rho}$. In this case, the cost function Eq. (4) can be modified into the following form:

$$C(q) = \| v - F\overline{\rho} - FWq \|_2^2 + \lambda \| q \|_2^2 \tag{5}$$

where $\rho = \overline{\rho} + Wq$, and the optimal solution is then given by

$$\rho = \overline{\rho} + \Theta F^H (F\Theta F^H + \lambda I)^{-1}(v - F\overline{\rho}) \quad \text{where } \Theta = WW^H \tag{6}$$

If the optimal solution is found for given W, W can be updated by taking p power of ρ. Through several iterations, the solution converges and the final solution becomes optimal solution. Additionally, by setting p to 0.5, we can easily verify that our k-t FOCUSS asymptotically solves L_1 minimization problem for ρ.

From Eq. (6), we can easily figure out that k-t BLAST is indeed the first iteration of out k-t FOCUSS algorithm except some detail facts.

First, our k-t FOCUSS is asymptotically optimal from the compressed sensing perspective. However, k-t BLAST which uses $p = 1$ does not minimize the L_1 norm, hence it is not optimal in compressed sensing perspective. Second, in k-t FOCUSS, the optimal solution can be achieved through iterations by updating W.

Result : To validate our algorithm, cardiac MR data is used. We have tested reconstruction performance with several acceleration factors. The reconstruction quality gets better as iteration increases and the motion artifact and spatial aliasing artifact are effectively removed even under 8 down sampling rate. Fig.1. shows the reconstruction results from 4x acceleration samples through 1 iteration and 5 iterations. We can figure that after 5 iterations the result gets sharp and detail structures like valves and vessels are clearly seen compared to after 1 iteration.

Conclusion : We have demonstrated that our k-t FOCUSS is asymptotically optimal from the compressed sensing theory point of view and clearly verified that k-t BLAST is the special case of k-t FOCUSS. Also, k-t FOCUSS has successfully applied for high resolution reconstruction of cardiac MR image even under severely limited samples.

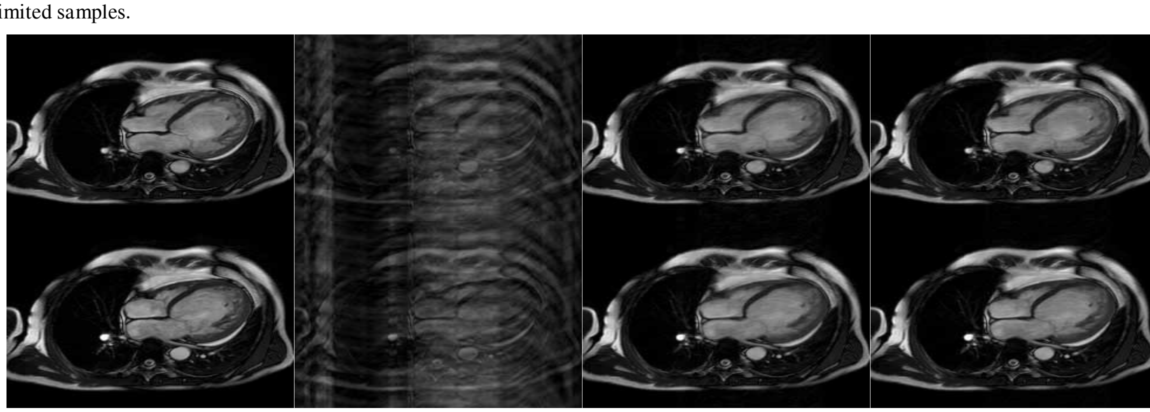

Fig1. (a) original image (b)zero-padded Fourier transform from measurement data (c) after 1 iteration (d) after 5 iteration

References

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