Iterative Reconstruction of Time-Resolved Projection Images Using Highly Constrained Back Projection

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Introduction: Angular undersampling decreases imaging time in projection MRI while maintaining image quality by oversampling the center of k-space [1]. However, large undersampling factors cause streak artifacts. By using a composite image reconstructed from nearly fully sampled data. Highly constrained backprojection (HYPR) [2] mitigates this problem for time resolved projection imaging in the absence of motion and for sparse data sets. For example, HYPR has been shown to perform well in time-resolved angiography where the data generally meet these conditions [2]. However, HYPR may not perform well in more spatially and temporally complex data as in, for example, perfusion data. Griswold et. al. have recently presented an iterative HYPR algorithm using the method of conjugate gradients to address these concerns. Here we present a novel iterative HYPR (I-HYPR) algorithm which uses the HYPR-reconstructed image as the composite image for the next iteration. This algorithm is mathematically similar to ordered subset expectation maximization (OSEM) [4,5] in which the initial guess image is the HYPR composite image.

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C =  R^-1[c]
For n timepoints:
  for i iterations:
    for k subsets:
          c =  R[ C ]
          m=  s / c
          M  =  R^-1[ m]
          C  =  C × M
    end k
  end i
  J_n = C
end n
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Figure 1: The iterative HYPR algorithm refines the composite, C, until it becomes the time point J_n.

Theory: In projection MRI with a time series the experiment is as follows: we make a series of n sinograms (s_n) of the same object by taking projections at sets of angles φ_n (note that each φ_n represents a set of angles covering 180°, which in general change with n). Our objective is to discover a set of images J_n which closely approximate the real images I_n given by, $\mathbf{R}\boldsymbol{\varphi}_\mathbf{n}[I_n] = s_n$, where $\mathbf{R}$ denotes the radon transform over the angles φ_n . Simple inversion of the equation can be achieved by using filtered back projection (FBP), however it is well known that severe artifacts result when the set of angles, φ_n, do not meet the Nyquist criterion. We use some or all of these sinograms, S_n, to create a series of composite sinograms (c_n). In general we require the composite sinograms to be sufficiently sampled to allow reconstruction by conventional methods such as FBP. We define the composite images, $C_n = \mathbf{R}^{-1}[c_n]$, where $\mathbf{R}^{-1}$ is the inverse radon transform using a suitable filter. The algorithm is outlined for the case of one subset in Fig. 1. Essentially the algorithm performs standard HYPR on a subset of the projections, updating the composite image for use on the next subset. Once all of the subsets have been done the process can be iterated until satisfactory convergence has been reached. Use of subsets speeds the convergence considerably as each subsequent subset 'sees' what the previous subset has done and can build on that information. This increases the number of effective iterations without significantly increasing the number of computations.

Simulation: For validation of the proposed algorithm, an extreme example was constructed consisting of 2 square objects in which one object disappears completely at a single time point (Fig. 2a). I-HYPR was performed on 3 projections of the time resolved image. Enlarged and cropped images of the first 5 iterations are given in Fig 2b. Note that the window and level differs from that in Fig. 2a so as to better visualize the residual error. After a single iteration (i.e. standard HYPR reconstruction), a substantial amount of residual signal error remains in the reconstructed time frame. To evaluate convergence the log mean signal value in an ROI placed in the upper object was calculated at each iteration and plotted in Fig. 2c. Note that in real imaging applications more than 3 angles will be used allowing the use of subsets for accelerating convergence. In the cerebral perfusion maps, the CBF maps were nearly identical for 32 projections and 16 subsets after 4 iterations (second to left in Fig. 3). The case of one iteration and one subset (far right, Fig. 3) depicts the performance of standard HYPR for this non-sparse data set.

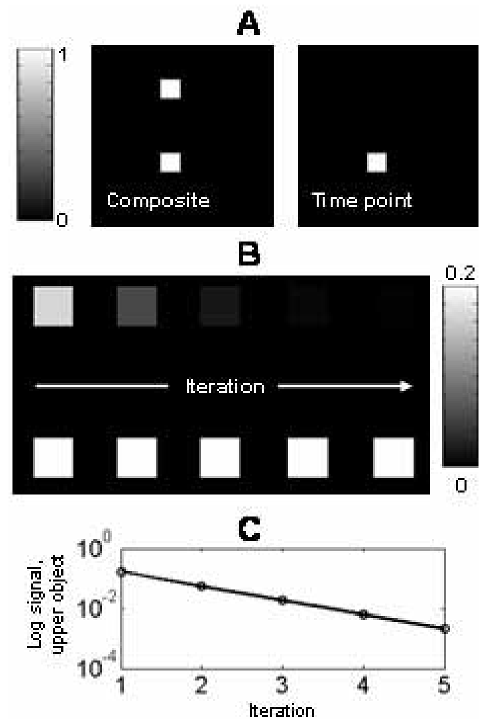

Figure 2: The simulated dataset is shown (A) above the five iterations of the I-HYPR algorithm (B) which have been enlarged and cropped. The mean of an ROI placed on the upper object is plotted (C) to show the convergence.

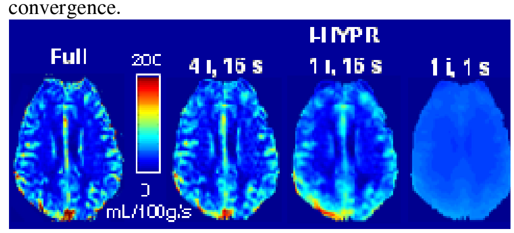

Figure 3: Cerebral blood flow maps reconstructed with HYPR (far right) and I-HYPR using 32 simulated projections.

	Full	4 i, 16 s	1 i, 16 s	1 i, 1 s
WM	33 ± 20	36 ± 19	29 ± 9	35 ± 0.5
GM	58 ± 22	64 ± 22	64 ± 17	42 ± 1

Table 1: Cerebral blood flow in mL/100g/s in white matter (WM) and grey matter (GM). The number of iterations performed and subsets used in each case are indicated by "i" and "s" respectively.

Discussion: In this simulation a new iterative HYPR algorithm is tested in simulation studies. The algorithm was shown to converge exponentially to the correct answer for computer simulated data. Feasibility for quantitative perfusion measures with substantial undersampling was also shown using simulations from human perfusion data. More study is needed to determine how applicable this technique is in real imaging setting including robustness to noise, motion and off-resonance conditions.

References: [1] Peters et al. MRM (2000). [2] Mistretta et al. MRM (2006). [3] Griswold et al. Int. MRA Club 18th Ann. Conf. 29 (2006) [4] Shepp and Vardi. IEEE Trans. Med. Imag. (1982). [5] Erdoğan and Fessler Phys. Med. Biol. (1999).

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