Fast Imaging By Using A Diagonal Covariance Matrix

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Introduction: Spatial redundancy of MR images has been widely exploited through techniques such as SMASH[1], SENSE[2], and GRAPPA[3]. For applications observing the motion of the region of interest over a period of time, however, temporal redundancy does not seem to have received a comparable attention; methods such as TSENSE[4] and UNFOLD[5] being previous works in the field. Viewing image acquisition as a random process was not considered before and it may yield a more robust and flexible method. In this abstract, we address the difficulties of this view and propose a framework that solves its problems. This framework can be applied whenever the reconstruction algorithm has a better response at high spatial frequencies. In-vivo images demonstrating the merit of the framework applied to Kalman filtering are also presented.

Theory: *Figure 1-a* shows the absolute value of the normalized autocorrelation function of an image taken from a real-time cardiac experiment. We observe that this function decays very slowly away from the origin. Therefore, the cross-correlations between pixels cannot be ignored if the autocorrelation function is to be utilized. This leads to an unstructured $n^2 \, x \, n^2$ covariance matrix for an $n \, x \, n$ image, thus presenting a tremendous computational load. On the other hand, if a circle around the origin of the k-space with an area of 1% of the total k-space support is masked out, image pixels become almost uncorrelated as shown in *Figure 1-b*. Therefore, the covariance matrix can be assumed to be diagonal, effectively reducing the size from n^4 to n^2. This way, huge savings are obtained in both memory and computational requirements.

The masked out region contains most of the image energy, but fully sampling such a small area around the origin is very inexpensive in MR imaging. The area is very small and the readout trajectory always starts at the origin. Thus, this region can be treated separately and the algorithm can be applied to the rest of the k-space with great efficiency.

Methods: The short axis of a volunteer is imaged using the RTHawk real-time architecture[6] with a four-interleaved spiral readout. The data is then separated into low and high spatial frequency components and a Kalman filter is applied to the high frequency part. The statistical estimates that the filter requires can be obtained either from the full data set or from a time window. A gradient echo sequence with a flip angle of *25°* is used for excitation so that the swirling pattern commonly seen in real-time spiral acquisitions does not appear at this low flip-angle regime. Note that this pattern would introduce cross-correlations at the high frequencies and hence has to be avoided.

Results: *Figure 2* shows two images, corresponding to the same frame of the real-time data. Both images are reconstructed using *1/4* of the full k-space.(one interleaf) This way, total imaging time becomes less than 25 ms. The image on the left is reconstructed conventionally and has disastrous aliasing artifacts, as dictated by the severe undersampling. The image on the right, however, is able to reconstruct successfully. The reconstruction uses the same amount of raw data, but the algorithm has access to statistical estimates and employs the technique described in this abstract. Notice that the right image is relatively noise free, considering the real-time acquisiton and the very short scan time.

Conclusions: We have shown that working with temporal statistics is feasible only over the high spatial frequency data. Our algorithm solves this problem and is able to reconstruct the underlying MR image using one-fourth of the raw data. This framework can be applied to other algorithms that have a preference for high spatial frequencies.

References: [1] Sodickson DK, Manning W, Magn Reson Med, 38:591-603, 1997
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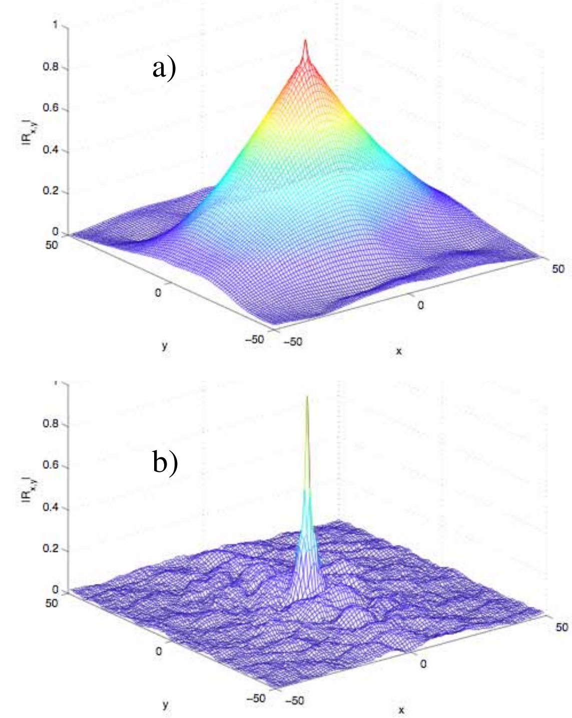

Figure 1 – Normalized absolute autocorrelation functions of a cardiac image – a) Full image, b) high frequency image obtained using a circular k-space mask. The autocorrelation functions are obtained from the power spectral densities by using Wiener-Khintchine theorem (assuming stationarity)

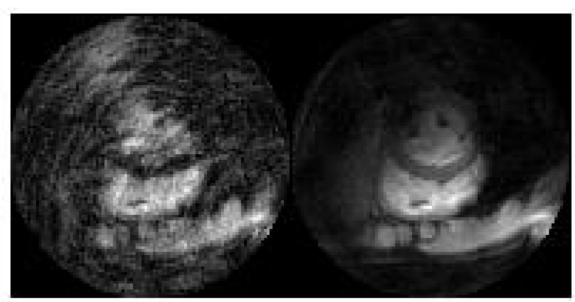

Figure 2 – Reconstruction using only one of four interleaves – Right: Conventional gridding, Left: Kalman reconstruction