A Statistical Approach to Non-Cartesian SENSE Regularization

B. Liu[1], L. Ying[1], G. Wu[2], J. Sheng[1], and S-J. Li[2]

[1]Electrical Engineering and Computer Science, University of Wisconsin-Milwaukee, Milwaukee, WI, United States, [2]Biophysics, Medical College of Wisconsin, Milwaukee, WI, United States

INTRODUCTION:

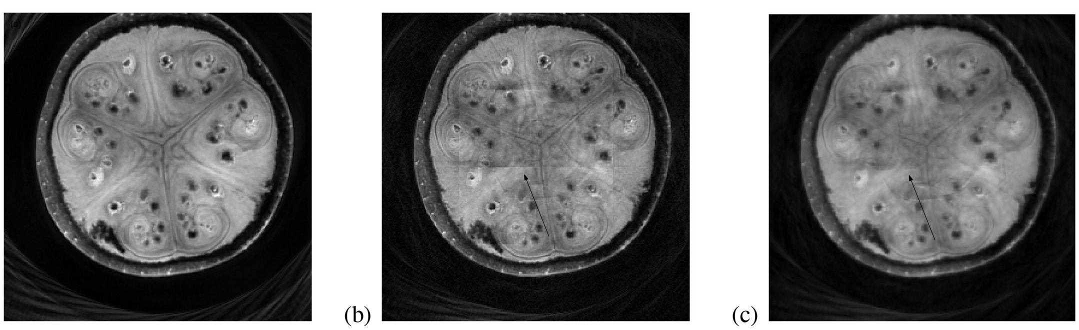
Parallel imaging using multiple receiver coils has emerged as an effective tool to reduce imaging time in various MRI applications. Although many advances have been made for Cartesian k-space trajectories, reconstruction methods for non-Cartesian trajectories still need further improvement. The iterative SENSE reconstruction method based on conjugate gradient (CG-SENSE) (1) has been widely used. It has inherent regularization effect to alleviate the ill-conditioning artifacts (2) because the low-frequency components of the solution tend to converge faster than the high-frequency components, and the number of iterations plays the role of regularization parameter. However, CG-SENSE has the so-called semi-converge behavior which means the results converge toward the exact solution first but diverge later (2). This undesirable property prevents SENSE reconstruction from further improvement through increasing the number of iterations. When a large reduction factor is used, the ill-conditioning problem becomes serious, the inherent regularization of CG is not sufficient to reduce the artifacts in the semi-converged results. To improve the reconstructed image for non-Cartesian SENSE, we propose a regularization technique based on the maximum a posteriori (MAP) estimation with the Markov random field prior. With little increase in computational complexity, the proposed method improves the convergence behavior and thus achieves a better reconstruction.

THEORY:

In the present of noise, both the desired image and the acquired k-space data can be treated as random fields. When Gaussian noise is assumed for the data, the conventional solution to the SENSE equation $m = \mathbf{E}v$ (with v the desired image, m the downsampled *k*-space data, $\mathbf{E}$ the sensitivity encoding matrix as in (1)) is equivalent to the maximum likelihood (ML) estimation of the image in a statistical framework. When the statistical distribution is given for the desired image, the MAP estimator is prefered to incorporate this prior information. The reconstructed image by the MAP estimator is given as $v = \arg\max_v \, p(m\,|\,v)\,p(v)$, where $p(m\,|\,v)$ is the likelihood function, and $p(v)$ is the prior distribution of the desired image. Here, we model the image as a Markov random field (MRF), which has been widely used to model medical images whose contextual dependence is primarily local (3). It has the flexibility to handle effectively both smooth and non-smooth features of an image (3). Under this model, the reconstructed image by the MAP estimator is given as $v = \arg\min_v \left(\| m - Ev \|_2 + \lambda V(v)\right)$ where V is the potential function of the MRF. A number of potential functions can be used. When a Gaussin MRF is used, it is equivalent to total variation regularization with L_2 norm: $v = \arg\min_v \left(\| m - Ev \|_2 + \lambda \| \nabla v \|_2\right)$ where ∇ denotes gradient. The regularizastion term assumes smoothness and penalizes the roughness of the reconstructed image. This regularization can decrease the condition number of the SENSE reconstruction and, therefore, speed convergence. We choose λ properly so as to not significantly degrade the spatial resolution relative to the natural resolution associated with the k-space trajectory. The above optimization problem is solved by the iterative conjugate gradient algorithm. To speed up the computation, nonuniform fast Fourier transform using min-max interpolation (NUFFT) (4) has been employed to calculate the sensitivity encoding onto a non-Cartesian trajectory.

METHOD AND RESULTS:

The raw MR data were collected on a GE 3T Signa scanner (Waukesha, WI) with an eight channel head coil and spin echo sequence (TE = 3.2ms, TR = 2sec, FOV = 24cm, matrix = 256*256, slice thickness = 5mm). The fully sampled data were acquired with 24 interleaves with 2332 points in each interleaf. We simulate the downsampled data with a reduction factor of 4 by keeping every 4 interleaves. All the algorithms were implemented in the MATLAB environment (MathWorks, Natick, MA). The performance of the proposed algorithm can be evaluated visually in Figure 1. Figure 1(a) show the sum-of-square image as the reference. Figure 1(b) shows the reconstruction from the proposed method after 10 iterations. For comparison, Figure 1(c) shows the CG-SENSE reconstruction without regularization also after 10 iterations. It can be observed that the reconstruction by the proposed method has less artifacts than the SENSE reconstruction, which is highlighted by the arrows in (b) and (c). The running time for both methods is about 4.7 seconds for each iteration.



(a) (b) (c)

Fig. 1: Reconstruction of a watermelon acquired with 8 coils, spiral trajectory, and $R = 4$. (a) SoS reconstructed from fully sampled data, (b) CG-SENSE without regularization, (c) Proposed reconstruction with $\lambda = 0.01$.

CONCLUSION:

A novel reconstruction method is proposed to address the semi-convergence problem for ill-conditioned SENSE reconstruction with non-Cartesian trajectory. The reconstruction is shown to have reduced artifact in comparison with the existing methods, when a large reduction factor is used.

REFERENCES:

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