Calculation of SNR degradation with the consideration of reference data

B. Zhang[1], C. Fu[2], J. An[2], Q. Zhang[2], and J. Wang[2]

[1]Siemens Mindit Magnetic Resonance Ltd., ShenZhen, Guangdong, China, People's Republic of, [2]Siemens Mindit Magnetic Resonance Ltd., ShenZhen, China, People's Republic of

Introduction

K-space based parallel imaging reconstruction methods [1] show very low artifact compared to image domain methods. An algorithm to calculate SNR degradation without consideration of reference lines was presented before [2]. Similar to the traditional SNR degradation analysis theory [3], that algorithm only considered the k-space uniform under-sampling case, i.e. pixels before iPAT reconstruction only aliased with the pixels apart from certain distances. Actually, k-space method (e.g. GRAPPA) does get additional SNR benefit from combining the reference lines to final k-space data [1]. In this abstract, a generalized SNR degradation evaluation algorithm was presented. Generalized iPAT encoding and decoding matrixes were exploited in this algorithm in order to analyze the contribution of autocalibrating sampling scheme. The SNR of autocalibrating SENSE can also benefit from this method.

$$\rho = \mathbf{C}\,\mathbf{F}\,\mathbf{G}\,r = \mathbf{M}\,r \quad (1)$$

$$SNR(w) = \frac{Signal}{Noise} = \frac{|w \cdot r|}{\sqrt{w \cdot \mathbf{\Psi} \cdot w'}\sqrt{mPE}} \quad (2)$$

Generalized SNR degradation theory

Assume we have an *nCh* array coil system with noise correlation matrix given by ψ. The number of phase encoding (PE) lines is *nPE* and the PE direction is along the column direction an in image.1D Fourier transform has been performed on the raw data in the readout direction. One column of the final image can be reconstructed using (1), where, *ρ* is a [*nPE*, 1] column vector, which denotes the *jRO*th column of final image. *r* is a [*nPE*× *nCh*,1] column vector, which denotes all the raw data at the *jRO*th readout from channel 1 to *nCh*. In *r*, missing data is filled by zero. Matrix **F** and **C** represent the processes of 1D Fourier transform in the PE direction and coil combination respectively. In **C**, a nonlinear sum-of-the-square (SOS) coil combination can be replaced by an equivalent linear combination using SOS profiles [5]. Matrix **G** denotes the k-space fitting, in which the autocalibrating sampling can be considered. **M** denotes the production of matrixes **C**, **F** and **G**. The dimensions of **M** are [*nPE* , *nPE*× *nCh*]. **M** can be called as generalized iPAT decoding matrix. If the *iPE*th row of **M** is defined as a row vector, $w_{jRO}^{iPE} = \left(w_1, w_2, \cdots, w_{nCh \times nPE}\right)$, w_{jRO}^{iPE} becomes the weighting coefficient of pixel index (*jRO*, *iPE*).

$$SNR^{rel}(w) = \frac{SNR(w)}{SNR^{opt}} \quad (3)$$

$$r = \mathbf{U}\,\mathbf{F}^{-1}\,\mathbf{P}\,\rho = \mathbf{E}\,\rho \quad (4)$$

The SNR per PE line of this array coil system with the weighting coefficient is given by (2). The matrix $\mathbf{\Psi}$ with dimensions [*nPE*× *nCh*, *nPE*× *nCh*] can be obtained by suitably disposing the elements of ψ. The signal and averaged noise amplitude are given by $|w \cdot r|$ and $\sqrt{w \cdot \mathbf{\Psi} \cdot w'}$ respectively. *mPE* denotes the measured PE lines. We use the Matlab (Mathworks, MA, USA) notation $\mathbf{H}'$ for complex conjugate transpose of the matrix **H**. The maximum available SNR of (2) is defined as the SNR of normally sampled, i.e. without parallel acceleration, images using the SNR optimized coil combination method [2] [4]. The relative SNR degradation due to parallel imaging can then be expressed by (3), where SNR^{opt} denotes the maximum available SNR. Also, SNR^{rel} represents the SNR efficiency relative to measurement time. So the maximum SNR efficiency is one.

Expression (3) can be used for any arbitrary array coil combination algorithm with the known weighting coefficients and sampling schemes. The concept, *generalized SNR degradation*, is used instead of reciprocal of g-factor, because the SNR reduction of k-space based reconstruction depends on not only the coil geometry but also the image content and sampling scheme. Yet the SNR degradation and reciprocal of g-factor are mathematically equivalent for sensitivity encoding.

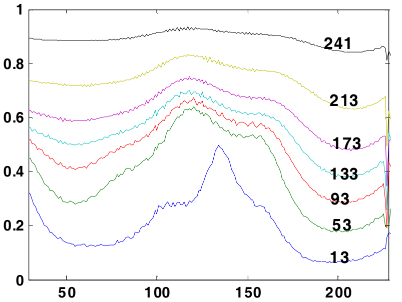

Fig.1 SNR degradation due to k-space method with combined reference data.

SNR Contribution due to combined reference lines

Phantom images were acquired on a Siemens Trio system using TSE sequence. Head-neck array and spine array were placed above and below the phantom. 256 PE lines were acquired with 2x parallel imaging acceleration mode. The coil system setup was chosen so that noise amplification for GRAPPA is very strong to show the effect. Fig.1 showed the SNR degradation of one image column in the readout center. Different numbers of reference lines, from 13 to 241, were used in order to perform k-space fitting and evaluate SNR reduction. When more and more reference lines were combined to the final image, the SNR increased and approximated to the SNR of normally sampled image.

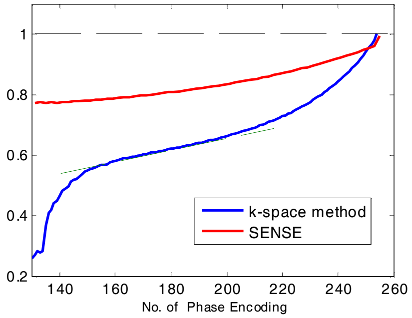

Fig.2 averaged SNR efficiency relative to number of measured PE lines of k-space method and SENSE

This method can also be applied to improve the SNR of autocalibrating SENSE. In original SENSE reconstruction, the sensitivity map is discarded after the unfolding weights are calculated. If SENSE encoding is expressed as a generalized encoding matrix, the final image SNR will benefit by inverting that matrix. In expression (4), $\mathbf{E}=\mathbf{U}^*\mathbf{F}^{-1*}\mathbf{P}$ denotes the generalized SENSE encoding matrix, where **P** and **U** denote the coil sensitivity matrix and the k-space under-sampling matrix respectively. If autocalibrating is not considered in **U**, inverting matrix **E** is equivalent to pixel-wise inversion of original SENSE mathematically but more time-consuming.

Fig.2 showed the averaged relative SNR of SENSE and k-space method relative to number of measured PE lines. When the number of reference lines is less than 13, the coefficients for k-space fitting is hard to be calculated precisely. So the coefficient calculated from 13 reference lines was employed in order to calculate the relative SNR. The results showed, when no reference line is combined, the algorithm is equivalent to the method presented by the authors before [2]. As the reference lines increased, the SNR efficiency also increased and approximated to one. The efficiency of k-space method is lower than SENSE, but also grows faster. From the slope, SNR efficiency curve is helpful to estimate the reference line efficiency. In this case, about 26 reference lines are the most efficient for imaging.

Conclusion

In this work, a new parallel imaging SNR degradation evaluation algorithm was presented. This algorithm employed generalized matrix notation to describe encoding and decoding of iPAT imaging. With the theory presented here, the SNR degradation with the consideration of reference data was shown. In addition, we pointed out that the SNR of SENSE imaging can be improved by using reference lines in the matrix inversion.

Reference

[1] Griswold, M.A., et al., *MRM*, 47,1202, 2002
[2] Wang, J, et al., Proc. Intl. Soc. Mag. Reson. Med 15 (2006) ; 3649
[3] Pruessmann, K.P., et al., *MRM*, 42, 952, 1999
[4] Roemer, PB, et al. The NMR phased array. MRM. 1990;16:192
[5] Wang, J, et al. Intl. Soc. Mag. Reson. Med 14 (2005) ; 242