Ultimate Intrinsic SNR of Regularized Parallel Imaging and Inverse Imaging

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INTRODUCTION

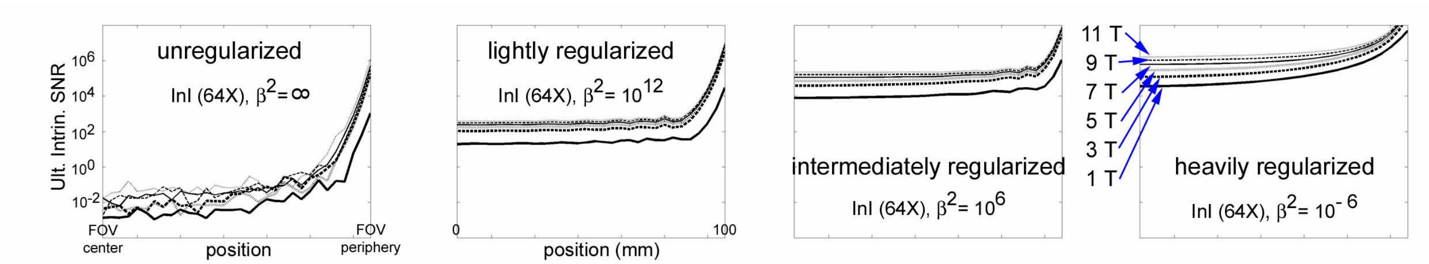
Parallel MRI using arrays of detector coils can increase the spatiotemporal resolution of MRI acquisitions using either k-space approaches or image-space approaches [1, 2]. MR inverse imaging (InI) has recently been introduced [3] as an extreme case of parallel MRI. Both parallel MRI and InI solve linear equations in the image reconstruction. Regularizing the reconstruction by incorporating prior information and/or improving the condition of the encoding matrix has been demonstrated to improve the SNR of the reconstructions. The upper-bound of SNR in traditional unregularized parallel MRI reconstruction has been previously reported [4, 5]. Here we investigate the theoretical upper bound, based on electrodynamics, of the intrinsic SNR in regularized parallel MRI and InI.

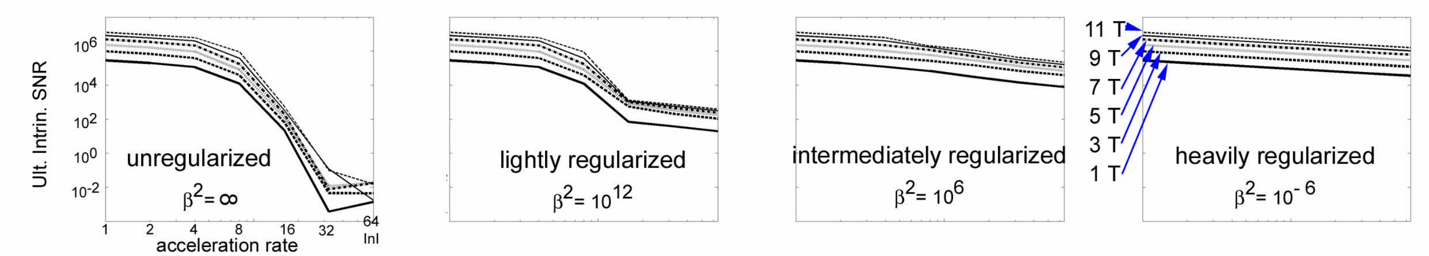
METHODS

In regularized parallel MRI and InI, a linear inverse operator **W** can be derived analytically: $\mathbf{W} = (\mathbf{A}^H\Psi^{-1}\mathbf{A}+\lambda^2\Psi)^{-1}\mathbf{A}^H\Psi^{-1}$, where **A** is the encoding matrix including both electromagnetic encoding functions (coil sensitivities or their equivalents for multipole basis functions) and gradient-derived Fourier encoding functions, Ψ is the noise covariance matrix and λ^2 is the regularization parameter. The superscript ^{H} indicates the complex conjugate and transpose. The ultimate intrinsic SNR at spatial location ρ can be written as: $\mathrm{SNR}(\rho) \propto \mathbf{R}_{\rho\rho} / \sqrt{[(\mathbf{W}\Psi\mathbf{W}^H)_{\rho\rho}]}$, where **R** is the resolution matrix $\mathbf{R} = \mathbf{W}\,\mathbf{A}$ [5]. The subscript $\rho\rho$ indicates the diagonal element of a matrix. Note that in traditional parallel MRI reconstruction using the weak SENSE constraint [2], **R** becomes the identity matrix. The choice of a regularization parameter was parametrically modeled as $\lambda^2 = \mathrm{Tr}(\mathbf{A}^H\Psi^{-1}\mathbf{A})/\mathrm{Tr}(\Psi)/\beta^2$, where Tr() is the trace of a matrix, and β^2 is a balance ratio. Ultimate intrinsic SNR calculations used a previously described multipole expansion into spherical harmonics in order to solve the Maxwell equations [5]. A spherical phantom at the iso-center was modeled, with 200 mm x 200 mm FOV bisecting the sphere perpendicular to the main magnetic field direction (i.e. in an axial orientation). This arrangement approximates an experimental setup for brain imaging. Dielectric constants and conductivity were varied using published frequency variations from 43 MHz (1T) to 468 MHz (11T). The β^2 was varied from 10^{-6} to 10^{12} and infinity in order to cover extremes of unregularized and heavily regularized scenarios.

RESULTS

Figure 1 (Top) shows plots of ultimate intrinsic SNR versus image voxel position at different β^2 and field strength in InI acquisitions (acceleration rate = 64). Peripheral image voxels and higher field give higher SNR. Decreasing β^2 (increasing λ^2) improves SNR, particularly from the unregularized case ($\beta^2=\infty$), which shows fluctuating SNR potentially due to the ill-conditioning in the unregularized InI reconstructions, to the lightly regularized case ($\beta^2=10^{12}$). **Figure 2 (Bottom)** shows plots of ultimate intrinsic SNR versus acceleration rate at FOV center. As expected, higher acceleration rate gives lower SNR due to signal loss and g-factor. Higher field improves the SNR at the same acceleration rate. Using regularization, SNR is generally improved, matching the trends observed in our previous reports [6]. Regularization especially increases SNR at high acceleration rates and for InI acquisitions.





DISCUSSION

In this work, we calculated the ultimate intrinsic SNR for regularized parallel MRI and inverse imaging. Concordant with previous reports for particular coil arrays, the ultimate intrinsic SNR drops at increased acceleration rates and decreased field strength. Using regularization, the trend of decreased ultimate intrinsic SNR can be mitigated. Without regularization, it is difficult even to achieve numerical convergence of the computations for higher acceleration rates and InI acquisitions. This is readily corrected using a light regularization, which also improves SNR by several orders of magnitude Heavier regularization yields higher SNRs. An appropriate choice of the regularization parameter is thus crucial to balance the SNR and the dynamic information, since a large regularization parameter is likely to suppress the measurement data in the reconstruction.

ACKNOWLEDGEMENTS

This project is supported by NIDA 1 R01 DA14178-0, NIH R01 HD040712, NIH R01 NS037462, NIH R01 EB002468, NIH P41 RR14075 and the MIND Institute.

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