A robust alternative to regularization in parallel imaging reconstruction

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Introduction

The total error in a parallel imaging reconstruction consists of two contributions: the reconstructed noise and the remaining image artifact (due to an imperfect reconstruction). Regularization is a means of trading these errors against one another. The choice of the regularization parameter is generally reported to be critical and its determination difficult and time consuming. Classical regularization approaches for parallel imaging reconstruction (Tikhonov, L-curve, truncated singular value decomposition) [1, 2] rely on the determination and exclusion or damping of small eigenvalues; depending on the applied norm, an optimal regularization parameter can be found. In this work another approach, involving iterative reconstruction, is analyzed [3]: it minimizes a cost function that relies exclusively on the immediate image characteristics of noise and artifact power. Image reconstruction amounts to minimizing a weighted sum of the two. It turns out that, in contrast to the reported difficulties with conventional regularization, the choice of the parameter (the relative weight of the two errors) is in fact uncritical in most relevant situations. The contribution of reconstructed noise and remaining image artifact to the final error is studied, as well as its dynamics over the iterations.

Materials and Methods

The reconstruction method discussed here relies on both noise covariance ψ and (the diagonal of the) image covariance θ and minimizes the total reconstruction error Δ_{tot} that is the weighted sum of the reconstructed noise and the remaining image artifact:

$$\Delta_{tot} = \alpha \cdot \Delta_{noise} + \Delta_{artifact}\,, \quad \alpha > 0, \quad \Delta_{noise} = \sqrt{F\psi F^H} \;\; and \;\; \Delta_{artifact} = \left\| (FE - T)A \right\|$$

where E is the encoding matrix , F the reconstruction matrix, T the target spatial response function (the intended product of FE) and A a linear mapping (e.g. with the corresponding signal estimates on the diagonal). This leads to the reconstruction formula ($^+$ denotes the Moore-Penrose pseudoinverse):

$$F = \alpha^{-1} T (\alpha^{-1} E^H \psi^+ E + \beta^{-1} \theta^+)^+ E^H \psi^+$$

In this work the two contributions to Δ_{tot} are analyzed for a large range of the weighting parameter α_{opt} and a variety of reconstruction cases. All the reconstructions were performed on a cluster of 44 computers, using CG iteration. The original image has 242x242 pixels and was acquired with 8 coils. From this input, MR-data was simulated on Cartesian as well as spiral trajectories, noise was added and then reconstructed iteratively over 4000 iterations. To compare reconstructed noise and pure image artifact, noise as well as noise-free data were reconstructed along with the actual data, such that the three reconstructions took the same path in CG-space.

Results

Most noticeably, the total error is essentially constant for a many orders of magnitude of alphas (all alphas below α_{opt}). The optimal reconstruction is only slightly better than the ones at lower alpha; this plateau of almost optimal reconstructions holds for many orders of magnitude. Moreover, the total image error is dominated by noise; this noise however might has some structure due to image-unfolding, but this structure does not come from imperfect unfolding. The situation changes for more challenging reconstruction problems (e.g. by increasing the reduction factor or decreasing the number of coils), where the problem gets ill-conditioned: the α_{opt}-dip is getting more pronounced, and thus choice of α more critical. The total error (at α_{opt}) is more and more dominated by the remaining image artifact. For α's below α_{opt}, the noise error shoots up and curbs the final image. Interestingly, an image of comparable error can be obtained for α's lower than α_{opt}if the reconstruction is stopped at the right number of iterations; this number however is not known a priori. During the CG-iteration, the noise power steadily increases while the image artifact steadily decreases; the relative dynamics decides whether the final error stabilizes or diverges. In challenging cases the noise tends to shoot up more than is won by a more faithful reconstruction whereas in simple cases the noise error stays bound since the freedom in noise amplification does not lead to a reduced image artifact. This leads to the fact that a decreasing remainder in the CG-algorithm can accompany an increasing total error. Therefore this remainder is a bad measure of the reconstruction quality. When reconstructing on non-Cartesian trajectories, it is important to apply a k-space filter [4], in order to prevent noise accumulation in the k-space edges that lead to diverging image error [5]. The choice of the image covariance (the ideal image covariance and a pure mask were compared) has only a minor effect on the reconstruction.

Conclusion

In case of reasonably well conditioned reconstruction problems, the choice of the regularization parameter α is non critical and the final error is dominated by noise. When the problem gets ill-conditioned, the choice of the correct alpha (or the right number of iterations) is critical and at α_{opt} the total image error is dominated by artifact. However, this case is of moderate practical relevance since the reconstructed image shows a large total error at any rate. Until convergence, the CG algorithm needs up to a few hundred iterations; reconstruction times are speeded up considerably by parallel computation. It seems important to note in this context that convergence of the CG algorithm ensures minimization of the weighted error sum but does not necessarily imply that the image artifacts converges to a minimum

References

[1] KF King, Proc.ISMRM01:1771 [2] FH Lin, MRM51:559-567 [3] J Tsao, Proc.ISMRM02:739 [4] KP Pruessmann, MRM46:638-651 [5] P Qu, Proc.ISMRM06:13

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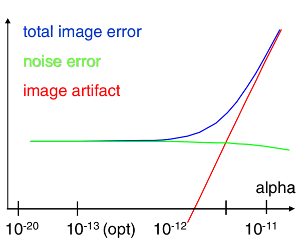

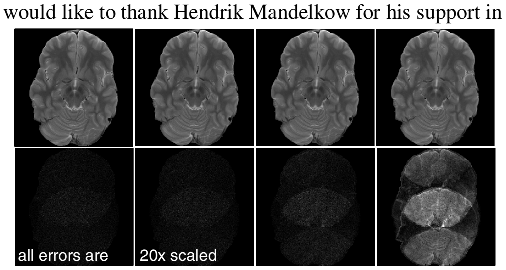

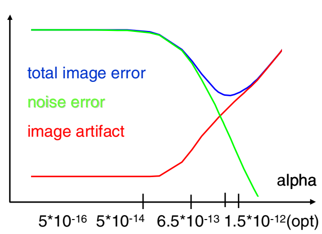

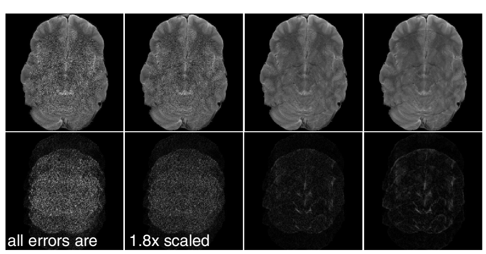

Fig1 (top left): Reconstructed images for reduction factor 3; below the optimal alpha, the reconstruction essentially remains identical wheras for larger alphas the image artifact quickly increases.

Fig2 (bottom left): Reconstructed images for reduction factor 7; below the optimal alpha, the reconstructed images degrade due to increase noise power. For large alphas the image artifact starts to dominate the total image error.

Fig3 (below): The total image error in function of alpha for the reduction factors 3 / 4 / 5 / 6 (from bottom to top). It can be seen that for increasing difficulty the dip grows.