Novel Numerical Modelling of Lumped Circuit Elements in RF Coils for Identifying Resonant Frequencies

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Introduction: Many different numerical approaches have been investigated in the past that are related to the topic of this research [1], but there seems to be an ever increasing need to model RF coils in real time, to better understand the field behaviour when an imaging sample is present. In this work the discrete numerical solution to Maxwell's equation is investigated, with the aim of modelling lumped element RF coils in real time, without the need to superimpose signals from different rungs or channels. Equivalent circuit theory when applied to RF coil design is somewhat limited, because it does not take account of the fact that RF coils are loaded when operational, and the consequent design using equivalent circuit theory cannot accurately predict any shifts on the resonant frequency and signal intensity losses that may be incurred due to the electromagnetic properties of the load. Past experiments have provided some interesting results, for example, RF head coils [2] and in parallel imaging at high field [3], but these are not without some limitations and modelling assumptions. The problem in modelling RF coils in terms of numerical approximations has to do with errors in the discretisation process. Errors such as lattice capacitance and inductance play important roles when numerical discretisation of the continuous Maxwell's equations is performed, and in this work, the aim is to considerably reduce these errors. The approach used here is not dissimilar to that used in antenna theory, for example [4, 5], and to some extent in circuit theory [6, 7]. The proposed numerical modelling approach aims to better simulate RF coils when a load is present (*e.g.* a head), whether it is a birdcage coil [2], or a multi channel arrangement [3]. When numerical models are used to predict certain phenomena, the simulations should provide sufficient data in time to allow for the prediction of indicators like field inhomogeneity (*i.e.* loss of intensity) and SAR levels.

Theory & Methods: In modelling RF coil phenomena the obvious choice of method in the numerical discretisation approximations is to use the full set of Maxwell's equations and use appropriate discrete models [1] to allow for the insertion of components, such as capacitors and voltage sources [6, 7], which are of primary importance in RF coil design. When taking a set of equations and applying a discrete template to them, the discretisation errors present themselves in the form of phase shifts and amplitude discrepancies. In terms of electromagnetic design, these can be considered in terms of lattice or mesh capacitances and inductances. In effect, the discretisation introduces capacitances and inductances, but due to the discretisation, step discontinuities between different electric and magnetic field components have been introduced unwittingly. These lattice capacitances (C_l) and inductances (L_l) are well known for the Yee discretisation of Maxwell's equations [8], and can be formulated given mesh criteria, and free space permittivity and permeability [9]. The mesh criteria Δ , which is defined as the space cell size in each coordinate direction, is proportional to both C_l and L_l . The problem with modelling RF coils at high frequencies is somewhat to do with the fact that the capacitor values required for insertion into the RF coil are in the order of 1pF to100pF. For standard discretisation approximations above 1 Tesla, the problem has to do with the value of C_l . For example, to capture the appropriate electromagnetic wave behaviour, Δ tends to be such that in the numerical models C_l is within one order of magnitude that of the capacitors to be inserted within the RF coil. This restricts the numerical modelling of the RF coil, because as the value of C_l approaches that of the inserted lumped circuit element, a subsequent shift on the resonant frequency is observed and no adequate conclusions about the numerical results can be made. Although it may seem at first that decreasing Δ is a simple solution, the problem lies in that halving the discrete cell size halves C_l , but increases the overall number of mesh points by a factor of eight in the case of a full 3D data set. This factor of eight is a large computational overhead, and best avoided, since Δ would generally have to be very small to model a whole body RF coil. This work presents an approach, in terms of numerical adjustments to the Yee cell [8], avoiding the need to drastically reduce the mesh size governed by Δ , while making sure that the lattice capacitance does not govern the numerical error in the simulations.

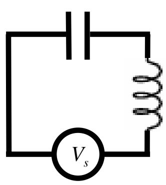

Figure 1. RF circuit.

Results & Discussion: In this work a single channel surface resonator is considered with equivalent circuit depicted by Figure 1, to demonstrate the ability of the modified FD-TD lumped element algorithm to compute resonant frequencies. The equivalent coil inductance is calculated using equations for self and mutual inductances [10] to enable prediction of capacitor values for resonance at relevant MRI frequencies. For the particular arrangement illustrated here, a number of capacitor values were used for the coil to obtain different resonant frequencies as depicted in Table 1. Figure 2 illustrates (a) the source voltage and (b) the resonant frequency as computed using the algorithm for the 70pF "old" simulation strategy. For the simulations a number of different parameters were chosen to illustrate the relative accuracy of the "old" approach [7] when compared to the "new" approach as outlined in this work. This work is not limited to simple resonators, but has been applied to body lumped element birdcage RF coils with dual voltage sources as well, and results for these will also be provided.

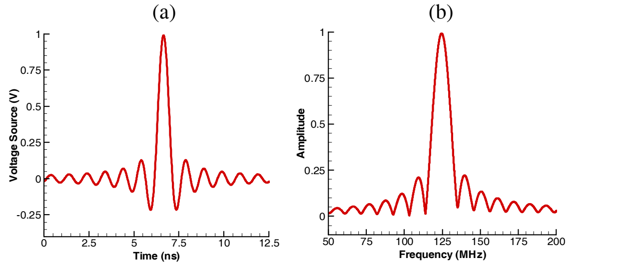

Figure 2. (a) Voltage source signal used for the simple RF coil and (b) envelope of the resonant frequency computed using the time simulation data.

Capacitor (pF)	Mesh	Analytic (MHz)	Old (MHz)	New (MHz)
20	41×41×41	237	148	261
30	31×31×31	204	137	191
40	41×41×41	168	130	168
50	31×31×31	156	128	169
60	41×41×41	137	121	148
70	31×31×31	133	125	146
80	41×41×41	118	115	132

Table 1. Illustration of the new approach when compared to the existing technique for different mesh sizes and different capacitor values.

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