Sphere model to study SENSE imaging performance

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Introduction. Design of coil arrays for Parallel Imaging is challenging due to the need to maximise array sensitivity and minimise noise amplification or g factor of the coil. The number of element coils necessary to be used for different applications without drastically affecting the image SNR is a very important issue. Arrays with a high number of coils represent a costly approach and demand a great deal of effort in their development. From the MRI literature, it seems that an indiscriminate number of coils should be used to get the best results. An expression of the SNR for SENSE as a function of the number of circular coil is derived to study this relationship. This theoretical derivation is based on the calculation of the magnetic field for a spherical conducting surface via a Lengendre differential equation. This model may serve to clarify how the SNR, is affected by the number of coils in an array, and give an idea about the optimal number of them for SENSE applications.

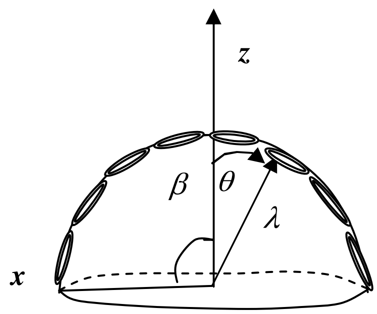

Figure 1. Setup of an array of circular coils around a conducting sphere.

Theoretical background. The classical electromagnetic theory establishes that the magnetic field for a spherical conducting surface with azimuthal symmetry (Figure 1), and a limited angular region, $0 < \theta < \beta$ can be derived using the Legendre differential equation:

$$\frac{d}{dx}\left[(1-x^2)\frac{dP}{dx}\right]+R(R+1)P=0 \qquad (1)$$

where $x = \cos(\theta)$ instead of θ itself, R (≥ 0) is taken as a parameter defining the number of lines in the k space (acceleration factor), and P is the magnetic field represented as a Lengendre polynomial.

The components of the magnetic field on the sphere [1]:

$$B_\lambda \simeq -RA\lambda^{R-1}P_R\cos(\theta) \qquad (2a)$$

$$B_\theta \simeq A\lambda^{R-1}P'_R(\cos\theta)\sin(\theta) \qquad (2b)$$

where the sphere radius is λ and P_R is a Lengendre polynomial and a function of the acceleration factor, R. From eqs. 2a & b and taking $\beta << 1$ and $R >> 1$, an approximate expression for R in this domain can be obtained from the Bessel function approximation,

$$P_R(\cos\theta) \simeq J_0((2R+1))\sin(\theta/2) \qquad (3)$$

Eq. (3) is valid for large R and $\theta < 1$. The first zero of $J_0(x)$ is at 2.405. This gives

$$R=\frac{2.405}{\beta}-\frac{1}{2} \qquad (4)$$

Eq. (5) can be rewritten to include the coil diameter and the sphere radius as follows,

$$R=\frac{2.405\lambda}{d}-\frac{1}{2} \qquad (5)$$

Therefore,

$$\frac{SNR_{SENSE}}{SNR_{FULL}}=\frac{1}{g\sqrt{\frac{2.405\lambda}{d}-\frac{1}{2}}} \qquad (6)$$

Eq. 5 is a function of the sphere radius (λ), and the circular coil diameter (d) for small angles θ. With this information is possible to determine the number of coils necessary to fully cover the sphere perimeter for SENSE applications. MATLAB programmes were specifically written to plot the number-of-coils-vs-SNR_{SENSE}/SNR_{FULL} (V. 6.1, The MathWorks, Natick, MA). The sphere radius was 10 cm for all calculations, and g =1,2,3 and 4 to simplify the computation of the SNR_{SENSE}/SNR_{FULL}.

Results and Discussion. The approach reported here shows that an SNR_{SENSE}/SNR_{FULL} expression as a function of the coil diameter can be derived from the classical electromagnetic theory with azimuthal symmetry and the Lengendre differential equation. The relative SNR_{SENSE}/SNR_{FULL} in eq. 6 shows a clear dependence of the coil dimensions and the size of the object to be imaged. A plot of number-of-coils-vs-SNR_{SENSE}/SNR_{FULL} using eq. (6) with a coil radius of 1 cm was obtained and shown in Fig. 2. As expected the SNR_{SENSE}/SNR_{FULL} grows with the number of the coils, as shown by the experimental evidence and the numerical simulations reported in the literature [3-4]. Unlike the results reported previously, this model is able to provide information on the performance of different coil element dimensions. It also shows how that there is a trade off between the SNR_{SENSE}/SNR_{FULL} and the g factor. It is clear that the geometry of the coil element can be changed and an equivalent SNR_{SENSE}/SNR_{FULL} expression can be derived. Then, This allows us to study the performance of a particular array of coils for applications where the object to be imaged has a form similar to a sphere. A simple model based on the classical electromagnetic theory was presented, and showed a great similarity with the numerical simulations and experimental results already reported. This approach can serve as an alternative to the trial-and-error approach to the development of array coils for SENSE applications, and saving a great deal of time and effort, and also to validate numerical simulations.

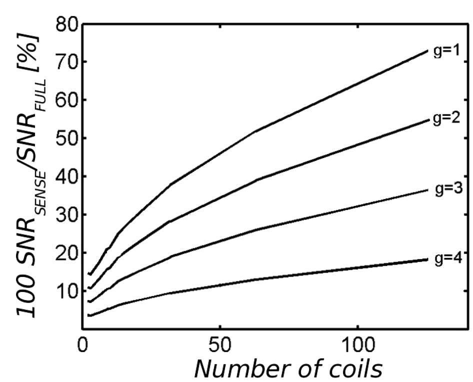

Figure 2. Plot of SNR_{SENSE}/SNR_{Full} vs # coils: SENSE performance is drastically affected by the g factor.

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