

# An Analytical Tool For Modeling High-Field RF Coils

X. Chen<sup>1</sup>, T. P. Eagan<sup>1</sup>, T. N. Baig<sup>1</sup>, and R. W. Brown<sup>1</sup>

<sup>1</sup>Department of Physics, Case Western Reserve University, Cleveland, OH, United States

## Introduction

In high-field MRI, wavelength effects lead to severe B1 field inhomogeneities [1]. To understand these spatial field oscillations, closed-form analytical solutions describing the rf fields in rf resonators have been presented [2-4]. In this work, we explore what we can learn from another analytical representation where the spatial oscillations are restricted to a single direction perpendicular to the imaging plane. This modeling can be compared to the so-called “TEM resonator,” which has been built to obtain homogeneous images in high field MRI [5].

## Methods

The oscillating magnetic field produced by a center-fed dipole antenna with finite length  $l$  carrying a sinusoidal standing wave current can be calculated analytically anywhere outside the antenna [6], giving the result

$$B_{\phi i} = -\frac{\mu_0 I_0}{4\pi j \rho} \left[ e^{-\beta \sqrt{\rho^2 + (z - \frac{l}{2})^2}} + e^{-\beta \sqrt{\rho^2 + (z + \frac{l}{2})^2}} - 2 \cos\left(\frac{kl}{2}\right) e^{-\beta \sqrt{\rho^2 + z^2}} \right]$$

where  $B_{\phi i}$  is the azimuthal component (the only component) of the magnetic field,  $I_0$  is the magnitude of the driving current,  $\rho$  is the radial distance to the wire,  $j = \sqrt{-1}$  and  $k = 2\pi/\lambda$  is the wave number. The antenna is aligned with the  $z$  direction with its center at the origin. Since a sinusoidal current vanishes at both ends of a center-fed dipole antenna [6], a “cage” structure can be constructed without any end-rings (Fig 1a). A pair of “parallel plates” can also be built up with dipole antennas of this kind (Fig 1b). The magnetic field produced by an array of antennas is the superposition of the fields of all individual elements. It is assumed for the present purposes that a dielectric with relative permittivity  $\epsilon_r = 60$  fills up the whole space. Therefore, the wavelength is  $\lambda = 9.68$  cm for 400 MHz ( $\sim 9.4$  T).

## Results and Discussion

B1 field profiles are shown in Fig 2 for conventional ( $\sin(\phi)$ ) birdcage models (and with no  $z$ -dependence along the legs) with a 25cm diameter and  $2.5\lambda$  ( $\sim 24.2$ cm) and  $10.5\lambda$  ( $\sim 1.02$ m) lengths, individually. Planar B1 field homogeneity in the central axial plane is significantly better for a longer cage ( $\sim 30\%$  variation within 7cm for a  $10.5\lambda$  cage,  $\sim 170\%$  for a  $2.5\lambda$  cage, as shown in Fig 3). The B1 field variation along the longitudinal direction in a  $10.5\lambda$  cage is close to  $\cos(kz)$ , while in the  $2.5\lambda$  cage it is not (Fig 3). These facts suggest that an approximate and expected uniform TEM mode exists in long cages. In fact, if the diameter of the cage is fixed and the length goes to infinity, a perfect uniform TEM mode would be set up as well as, all the higher order modes. Similar results exist for “parallel plates” built up with dipole antennas. For “parallel plates”, a smaller distance between two plates also provides better uniformity (the width must not be small). For a center-fed dipole antenna, the magnetic field falls as  $1/r$  radically in a small “near zone” around the antenna, and then starts oscillating. As the length of the antenna increases, the size of the “near zone” where the field shows static features, rather than oscillations, grows. Therefore, the geometry of the antenna array, especially the length, has a remarkable effect on the magnetic field it produces. On the other hand, since dipole antennas of any length can be considered to be a linear combination of shorter dipoles with independent oscillating sources, in particular, half-wavelength long dipole antennas, certain configurations of these small sources may produce special magnetic field distributions (for example, a uniform field). Here, we provide analytical models for waveguides which support a uniform TEM mode (at least approximately). The modeling has been applied to shielded systems, which more closely resemble the so-called “TEM resonator” where fields are concentrated between the strut and shield. Note the contrast between these results and the “classical birdcage” with its constant current profile in the  $z$ -direction.

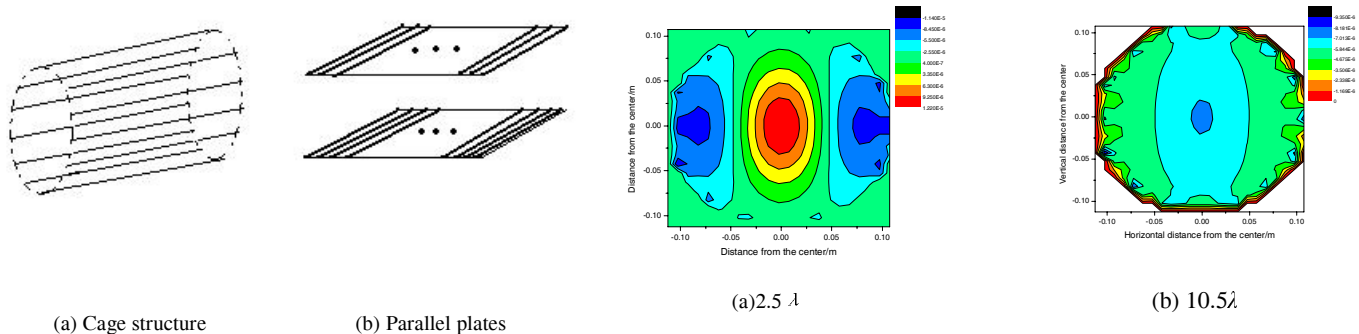


Fig 1 Arrays of dipole antennas with different geometries

Fig 2 Contour plots of the y component of the B1 field in the central axial plane for birdcage-like antenna arrays with different lengths.

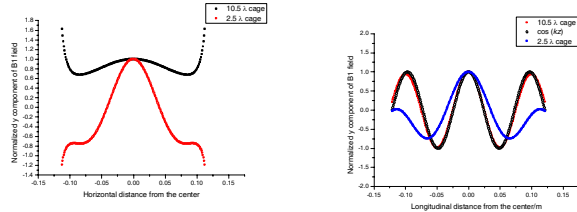


Fig 3. Uniformity along the horizontal axis in the central axial plane (left) and the field variation along the longitudinal direction (right) for birdcage-like antenna arrays with different lengths.

## References

- [1] Vaughan JT, et al. Magn Reson Med 46: 24-30 (2001)
- [2] Tropp J, J Mag Res 167: 12-24 (2004)
- [3] Foo TKF, et al. Magn Reson Med 21: 165-177 (1991)
- [4] Foo TKF, et al. Magn Reson Med 23: 287-301 (1992)
- [5] Vaughan JT, et al. Magn Reson Med 32: 206-218 (1994)
- [6] Balanis CA, Antenna Theory: Analysis and Design, Harper & Row, New York, 1982

**Acknowledgement** We are grateful to Shmaryu Shvartsman and Victor Taracila for discussions. This work is supported by the Ohio Third Frontier Initiative.