Basis Selection for Wavelet Processing of fMRI Data

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Introduction

Wavelet-based methods for contrast detection and denoising of functional magnetic resonance imaging (fMRI) data have gained popularity [1,2,3]. The primary motivation for using wavelets is that a wavelet basis can sparsely represent a wide class of signals. Although detailed analyses of wavelet-based methods for fMRI have been performed [1,2,3], little attention has been given to the selection of a suitable wavelet basis. The sparsity of a wavelet representation of a signal depends on both the wavelet basis (determined by the mother wavelet, the number of decomposition levels, and the choice of using a dyadic or an overcomplete expansion) and the exact form of the signal. Activation in an fMRI dataset is inherently sparse, with some activation regions being large (e.g. primary cortex) and others being small (e.g. hippocampus). Suitable selection of a wavelet basis for fMRI data must incorporate this sparsity. This principle is demonstrated for wavelet denoising of fMRI data in the hippocampus.

Wavelet Basis Selection Criteria for fMRI Data

Let **s** denote an fMRI dataset that has been processed to remove the mean of each time course, leaving only functional data corrupted by (approximately) Gaussian noise [1]. Furthermore, assume that **s** has M features (true areas of activation) that can each be described by an R-order polynomial. Denote by $\mathrm{FS}_{m,d}(\mathbf{s})$ the size (in pixels) of the m[th] feature of **s** in dimension $1 \le d \le D$, where D is the number of spatial dimensions.

Consider a J-level dyadic or overcomplete (undecimated) wavelet expansion of **s** using the length-L scaling and wavelet functions ψ and ϕ, respectively. Let **S** denote the wavelet-domain signal. In order for the basis corresponding to the wavelet expansion defined by ψ, ϕ, and J to be preferable over the basis of the sampled data, **S** should be at least as sparse as **s**. To be conservative, we will assume that during the wavelet expansion no features merge. It can be shown that to ensure **S** is at least as sparse as **s**, the dyadic or overcomplete wavelet basis inequalities shown to the right must be true for all values of d. These inequalities describe the maximum filter length (L) and maximum number of decomposition levels (J) for a dyadic or overcomplete wavelet expansion. L must also be large enough to allow for at least R+1 vanishing moments. In practice R=0 can often be assumed.

Dyadic Wavelet Basis Inequalities

$$L \le \frac{\frac{1}{M}\sum_{m=1}^{M}\mathrm{FS}_{m,d}(\mathbf{s})\left(1-2^{-J}\right)}{2J-\left(1-2^{-J}\right)}$$

$$\frac{2J}{1-2^{-J}} \le \frac{1}{ML}\sum_{m=1}^{M}\mathrm{FS}_{m,d}(\mathbf{s})+1$$

Overcomplete Wavelet Basis Inequalities

$$L \le \frac{\frac{J}{M}\sum_{m=1}^{M}\mathrm{FS}_{m,d}(\mathbf{s})}{5(2^{J}-1)-2J}+1$$

$$\frac{2^{J}-1}{J} \le \frac{1}{5}\left(\frac{\sum_{m=1}^{M}\mathrm{FS}_{m,d}(\mathbf{s})}{M(L-1)}+2\right)$$

Methods

Rest fMRI data was obtained by acquiring T2*-weighted gradient-echo, echo-planar imaging data (TR/TE = 3000/25.3 ms, 90° flip angle, 64x64 matrix, 26 slices, 130 volumes) on a 3T whole-body MR scanner (GE Healthcare) while a healthy subject fixated on a white cross for the entire acquisition. Twenty-five rest datasets were created by resampling these acquired data using the wavestrapping resampling technique [4]. Synthetic block (30 sec active/rest) activation was added in a single 4x4 region near the right hippocampus with 0.5% contrast. The mean of each time course was removed from the resulting dataset. Each image was then denoised using three different wavelets (Haar, length-4 Daubechies, and Battle-Lemarie degree-1 spline) and a soft threshold. The size of activation dictates that an overcomplete wavelet expansion be used with a single decomposition level and a maximum filter length of 2.33. A receiver operating characteristic (ROC) curve was computed from each denoised dataset.

Functional MRI (TR/TE=2500/25.3 ms, 90° flip angle, 64x64 matrix, 26 slices, 110 volumes) was performed for pre-surgical planning on a subject with a right hemispheric arteriovenous malformation. The clinical question is whether function (activation) in the left hippocampus. An MR-compatible synchronization control system (MRIx Technologies, Bannockburn, IL) was used to present a visual block design (25 sec active/rest) memory paradigm to the subject. The image data was denoised as described above using the three different wavelet bases. No other corrections were performed. Contrast detection was performed on the original and denoised data using a standard t-test at an uncorrected threshold of t=4.75. This threshold was selected to allow activation in the left hippocampus to be seen in the original (non-denoised) data. The activation of the left hippocampus was verified using a second functional paradigm based on language comprehension.

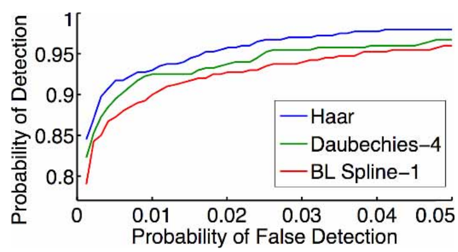

Fig. 1: Average ROC curves after wavelet denoising

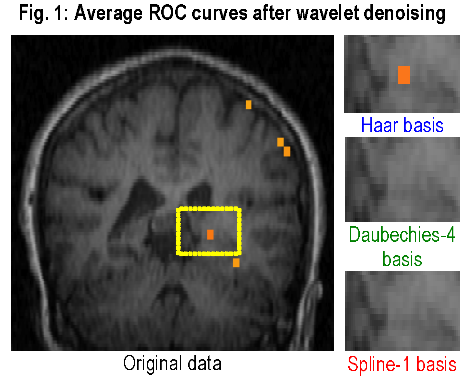

Fig. 2: Contrast detection results on original and wavelet denoised data. The yellow region of interest box indicates the spatial location of the small images.

Results

Fig. 1 shows the average ROC curve after denoising the synthetically activated rest data using each wavelet basis. As expected from the overcomplete wavelet basis inequalities, the Haar basis provides the best denoising performance. The other two wavelet bases perform less well as their filter length exceeds the maximum filter length inequality.

Fig. 2 shows the result of performing contrast detection on the pre-surgical data before and after wavelet denoising. Activation in the left hippocampus is detected in a single voxel of the original data. After wavelet denoising this activation can only be detected (at the same threshold of 4.75) if the Haar wavelet basis is used for denoising. Based on the filter length inequalities and the fact that the hippocampus is known be small (and is therefore represented by a small number of voxels) this result is expected due to the Haar basis having the shortest filter length.

Conclusion

The inherent sparsity of functional data should be considered when selecting a wavelet basis for processing fMRI data. Although many wavelet bases may be suitable when the primary interest is large features, a limited subset of those bases provides a sparse representation of fMRI data with small features. To form the best estimate of small regions of activation using a technique such as wavelet denoising, a wavelet basis with very short filters should be used. Although (non-Haar) symmetric wavelets are generally preferred due to their linear phase property, such wavelets are typically longer than non-symmetric wavelets with comparable number of vanishing moments and therefore are not applicable for detection of small regions of activation.

References

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