Calculation of Aorta Pressure Waveform from MRI Blood Flow Velocity Measurement

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Introduction: Aortic pulse pressure is an important clinical parameter. It can be measured invasively by catheterization or estimated by brachial cuff pressure. However, brachial pressure is not a consistently accurate estimate of central aortic pressure [1]. Phase contrast magnetic resonance imaging (PC-MRI) can accurately measure aortic blood flow and pulse wave velocity. According to the mathematical model presented here, blood flow velocity and pressure are related by two differential equations. Using PC-MRI blood flow data as input, with proper boundary conditions, the aortic pressure waveform at any specified region of interest (ROI) can be generated from the mathematical model.

Theory: This method is based on a one-dimensional model considering both the Windkessel effect and the traveling wave properties of pulsatile blood flow in the aorta [2]. Similar to the Windkessel model, this model has an electrical circuit analogy, which has a uniform distribution of capacitors and resistors along two parallel conducting lines. Therefore, telegraph equations are proposed to describe the relation between blood flow velocity and pressure. With the additional assumption that aortic cross-sectional area A is a function of pressure P only [3], the telegraph equations can be derived from linearized one-dimensional Navier-Stokes and continuity equations as:

$$\frac{\partial P(z,t)}{\partial z} = -\rho \frac{\partial u(z,t)}{\partial t} - Ru(z,t), \qquad \frac{\partial u(z,t)}{\partial z} = -C_p \frac{\partial P(z,t)}{\partial t} - GP(z,t), \tag{1}$$

Where u is the blood flow velocity waveform; ρ is the blood density; $C_p = (\partial A/\partial P)/A$ is the aortic compliance; R is the viscous damping coefficient; and G is the leaking conductance. Since R and G are small, we neglect them to calculate the pulse wave velocity v:

$$v = \frac{1}{\sqrt{\rho C_p}} \tag{2}$$

Same as solutions in telegraph equations, u and P are related by impedance Z_ω:

$$Z_\omega = \sqrt{\frac{R + i\omega\rho}{G + i\omega C_p}} \tag{3}$$

The dispersion relation is:

$$k^2 = (R + i\omega\rho)(G + i\omega C_p) \tag{4}$$

Since blood flow is partially reflected by the iliac bifurcation, we adopt a partial reflection boundary condition. We assume that there is only one reflection site somewhere in the distal aorta, but the exact location of reflection site is not important. Therefore, the blood flow velocity waveform u_0 measured by PC-MRI is a superposition of incident velocity waveform u_i and reflected velocity waveform u_r with $u_r(t)=ru_i(t-\tau)$ (r is the reflection coefficient, τ is the time delay). The complex side-wall boundary condition is avoided. Using plane wave expansion, the incident and reflected velocity waveforms can be separated. The relation between blood flow velocity and pressure is:

$$P(z,t) = \sum_\omega Z_\omega u_i(k) e^{ikz} + Z_\omega u_r(k) e^{-ikz}. \tag{5}$$

Our method requires only two blood flow velocity measurements as input. From one aorta para-sagittal view with in-plane one-dimensional velocity encoding along the primary direction of flow, u_0 and v are measured. Then we can calculate C_p according to Eq. (2). A second measurement is made in the common carotid artery axial view with through-plane one-dimensional velocity encoding. Typically, there are two velocity peaks, corresponding to incident and reflected velocity waveforms. Using the time delay between two peaks, v, and the location of ROI, τ can be calculated, and r can be estimated from the ratio of two peak heights. The dispersion effect is neglected. After assuming that the branching is uniformly distributed in aorta, we can estimate G which is very small comparing to ωC_p. Impedance phase angle (comes from viscoelasticity of vessel wall) is empirically adjusted to 15 degrees to determine R, the only free model parameter. Using the parameters measured or calculated above, u_i, u_r, and Z_ω are calculated. According to Eq. (5), we get pressure waveform P at ROI.

Methods: The experimental study was done on a 1.5T MRI system (Avanto, Siemens, Germany) in the Ross Heart Hospital at The Ohio State University. Four patients (age range from 27 to 76 with mean age 50) who had both MRI and catheter-based pressure measurements (three on the same day and one five days before) were included. The blood flow velocity waveforms were measured in aorta and carotid arteries and the corresponding aortic pressure waveform was calculated. The imaging parameters were: 192 x 144 or 192 x 120 matrix, 5.0mm thick slice, flip angle 15 or 25 degrees, TR =14 ms, TE = 3.1 ms, pixel bandwidth=355 Hz.

Results: Mean aortic pressures measured by a fluid-filled catheter system, averaged over 5 cardiac cycles vs. estimated mean aortic pressures using the mathematical model in four patients are summarized in the Table. Compared to direct pressure transduction, the aortic pressure waveforms generated by the model are of similar shape and amplitude (Figure).

Table

subject	1	2	3	4
Mean (mmHg)	55.3	35	35.1	37.5
SD (mmHg)	3.5	3	4.9	2.9
Model Calculation (mmHg)	52.7	35.6	31.9	40.8

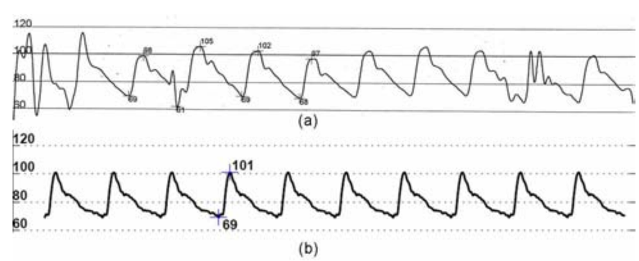

Figure. Pressure waveform (mmHg vs. time) directly transduced in the ascending aorta via fluid-filled catheter system (a) versus aortic pressure waveform generated through model calculation in a 42 year-old male who underwent CMR and cardiac catheterization on the same day.

Discussion: Based on a mathematical model describing the blood flow velocity and pressure relation in aorta, a new method was developed to calculate blood pressure from blood flow velocity measured by PC-MRI. This new method allows noninvasive quantitative measurements of pulse pressure and qualitative estimates of aortic pressure waveform. Future work will be to include nonlinear terms and to collect additional validation data.

References: [1] Pauca A.L., et al, Chest 102(4): 1193-1198, 1991. [2] Anliker, M., et al., Z. Ang. Math. Phys. 22: 217-246, 1971. [3] Westerhof N., et al., J. Biomechanics 2(2): 121-143, 1969.