Fast and Super Fast Estimation of Quantitative Parameters in DCE-MRI

C. Yang[1], G. S. Karczmar[2], and W. M. Stadler[1]

[1]Department of Medicine, The University of Chicago, Chicago, IL, United States, [2]Department of Radiology, The University of Chicago, Chicago, IL, United States

Introduction: In Dynamic Contrast Enhanced MRI (DCE-MRI), semi-quantitative parameters such as the Area Under the Curve (AUC) (1) are commonly used as endpoints; quantitative parameters are also generally felt useful such as the contrast agent (CA) transfer rate K^{trans}. Quantitative parameters are obtained by modeling the CA dynamic data with a kinetic method, for instance the "Tofts model with the blood plasma volume v_p term" (2) which was preferred by several recent DCE-MRI consensus recommendations. This model describes the CA concentration vs. time curve $C_t(t)$ as: $C_t(t) = v_e k_{ep} \int C_p(\tau)\exp[-k_{ep}(t-\tau)]d\tau + v_p C_p(t)$ Eq.[1],

where v_e is the extracellular extracellular space (EES) volume, $k_{ep} \equiv K^{trans}/v_e$ represents the rate constant of CA backflux from the EES to the blood plasma, $C_p(t)$ is the arterial input function (AIF). Traditionally those quantitative parameters are estimated by the least square fitting (**LSF**) method, which is often time-consuming because numerous iterations are needed and the convolution term in those models is computationally expensive. For AIFs in certain analytical forms, the convolution can be done explicitly; for AIFs in general form the Fast Fourier Transform (FFT) algorithm is often used to perform the convolution.

Materials and Methods: Because the convolution term in Eq.[1] has an exponential kernel, the following holds:

$$f(d_j) = \int_0^{d_j} C_p(\tau)\exp[-k_{ep}(d_j-\tau)]d\tau = \exp[-k_{ep}(d_j-d_{j-1})]\cdot\int_0^{d_{j-1}} C_p(\tau)\exp[-k_{ep}(d_j-\tau)]d\tau + \int_{d_{j-1}}^{d_j} C_p(\tau)\exp[-k_{ep}(d_j-\tau)]d\tau$$
$$\cong \exp[-k_{ep}(d_j-d_{j-1})]\cdot f(d_{j-1}) + C_p(d_j)\cdot\{1-\exp[-k_{ep}(d_j-d_{j-1})]\}/k_{ep} \quad \text{Eq.[2]},$$

where d_j $\{j=1,...,D\}$ are the time grids, and $f(d_j)$ is defined as the integral over $[0, d_j]$. Starting from j=1, one by one in a sequential order the next integral $f(d_j)$ can be done quickly using the previously calculated $f(d_{j-1})$ according to Eq.[2]. With this "*fast sequential convolution method*" (**FSCM**), the computational time to calculate the convolutions at all the time grids is proportional to D. In contrast, the computational time of the FFT is much longer - proportional to $2(2D)Log_2(2D)$ because zero-padding and two FFT have to be used. For example the FFT is 40 times slower when D=512. In Eq. [2] a Euler rule is used for the $C_p(\tau)$ term in the numerical integral over $[d_{j-1}, d_j]$, alternatively higher order approximations such as a trapezoidal rule can be used.

Here we also present a **LACK** algorithm which gives a super fast estimation of the quantitative parameters by directly **L**inking the **A**rea under the **C**urve parameters with the **K**inetic parameters. P_i and N_i respectively denote the AUC of the $C_p(t)$ and $C_t(t)$ at time interval i. For the Tofts model in Eq.[1], three time intervals i=1, 2, 3 are needed – for example we can respectively select them as [0s, 45s], [45s, 90s] post bolus arrival and the last 120s. Step by step, we calculate k_{ep}, v_p / v_e and v_e using those AUC parameters. Denote $m_i = N_i / N_3$ and $d_i = P_3 N_i / N_3 - P_i$, there is a relation: $\hat{N}_1(k_{ep})d_2 - \hat{N}_2(k_{ep})d_1 - \hat{N}_3(k_{ep})(P_1d_2 - P_2d_1) = 0$ Eq.[3], where we use $\hat{N}_i(k_{ep})$ to denote the AUC of time interval i for the simulated noise-free $C_t(t)$ at k_{ep}, v_e=*1* and v_P=*0*. Based on Eq.[3] we can create a look-up table of $\hat{N}_i(k_{ep})$ at different k_{ep}. At step 1, we use the look-up table and linear interpolation to estimate k_{ep}. In step 2, using the estimated k_{ep} we calculate $v_e / v_p = [\hat{N}_1(k_{ep}) - \hat{N}_3(k_{ep})m_1] / d_1$. In step 3, we calculate $v_e = N_3 / [(v_p / v_e)P_3 + \hat{N}_3(k_{ep})]$. Finally, we obtain $K^{trans} \equiv k_{ep} v_e$ and v_p.

We evaluate the two algorithms using clinical DCE-MRI data acquired from renal cancer patients at 2s resolution for about 7min post injection of 0.1mmol/kg Omniscan. The AIF was obtained with a multiple reference tissue method (3). The kinetic parameters are estimated with the LACK method, and compared with the results obtained by the traditional LSF method where the FSCM is used for the convolution. The Levenberg-Marquardt approach is used for the minimization in the LSF. The software is written in IDL (RSI) and ran on a PC (Dell, 2.8GHz).

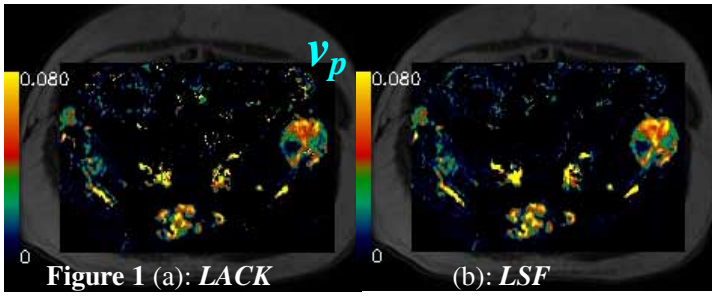

Figure 1 (a): ***LACK*** (b): ***LSF***

Results: Equipped with the FSCM, the speed of the traditional LSF averages about 140 voxels/s. The speed of the LACK is much faster, averaging about 90,000 voxels/s. In the representative example shown in Fig. 1, the rectangular ROI has about 23,000 voxels, which took less than 0.25 second to compute with the LACK. As shown by the v_p maps in Fig. 1, parameter maps obtained by the LACK were very similar to those by the LSF, but they are slightly noisier.

Discussions and Conclusions: With the help of the FSCM, quantitative parameters can be estimated with the traditional LSF at fast speed. The LACK provides a super fast and robust estimation of quantitative parameters, computing an entire slice in sub-second. The LACK further shows that there is a direct connection between the quantitative parameters and the semi-quantitative AUC parameters (1).

References: **1**. Evolhoch JL. JMRI 1999; 10: 254-259. **2**. Tofts PS, et al. JMRI 1999; 10:223-232. **3**. Yang C, et al. Proc. ISMRM, 2006, p384.