Reconstruction of variable-density data in Fourier velocity encoding

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Introduction: Fourier velocity encoding (FVE) [1] is useful in the quantitation of valvular stenosis and regurgitation, as it eliminates partial-volume effects that may cause loss of diagnostic information in phase-contrast imaging [2]. Velocity resolution (v_{res}) can be improved using variable-density sampling along the velocity dimension (k_v) [3], but considerable aliasing artifacts arise when a density reduction factor (ρ) greater than 2 is used with conventional reconstruction methods such as gridding [4] and DrFT [5]. We propose k_v-FOCUSING (FOV Centering Using Sinc INterpolation), a reconstruction scheme that eliminates artifacts caused by variable-density sampling for typical valvular flow distributions. Phantom and *in vivo* validation was performed using MR Doppler [3] and spiral FVE [6,7] acquisitions.

Theory: k_v-FOCUSING combines variable-width sinc interpolation along k_v with velocity field-of-view (v_{FOV}) centering. Sinc interpolation is computationally feasible because FVE datasets are typically small. Without the need for deapodization, a different kernel-width for each k_v sample can be used. Variable-width interpolation [8] reduces artifacts by trading v_{res} for v_{FOV}, which is desirable as plug flow contains high-frequency components in k_v, but occupies only a small portion of the v_{FOV}, whereas flow jets fill a wider portion of the v_{FOV} but require lower v_{res}. Because this approach maximizes v_{res} at the center of v_{FOV}, the v_{FOV} is shifted towards the center of the distribution by applying a linear phase to the sinc kernels based on an estimate of the average velocity. An overgrid factor of 1.5 is used, and the reconstruction is performed independently for each time-frame.

Let $S(k_v)$ be the Fourier transform of the velocity distribution $s(v)$, and S_n be a sample of $S(k_v)$ taken at $k_v = \kappa_n$. Then conventional gridding can be represented by $\zeta(k_v) = \sum_n w_n S_n \Psi(k_v - \kappa_n)$, where $\zeta(k_v)$ is an estimate of $S(k_v)$, $\Psi(k_v - \kappa_n)$ is a constant-width convolution kernel (e.g. Gaussian, Kaiser-Bessel), and w_n is the corresponding weight of each sample, calculated as its support region, i.e. $w_n = ½(\kappa_{n+1} - \kappa_{n-1})$. k_v-FOCUSING eliminates undersampling artifacts by: 1) **utilizing a variable-width sinc kernel $\Psi_n(x) = w_n^{-1}\,\mathrm{sinc}(x/w_n)$**, and 2) **applying a linear phase to center the v_{FOV}**. A Hamming window is applied to reduce ringing artifacts. k_v-FOCUSING can be written as:

$$\zeta(k_v) = \sum_n w_n S_n \Psi_n(k_v - \kappa_n)\, hamm(\, k_v - \kappa_n / \max|\kappa|\,)\, \exp(-j\, 2\pi\, (k_v - \kappa_n)\, v_o)$$

where $hamm(x) = 0.54 - 0.46\cos(\pi x - \pi)$ for $|x| \le 1$, 0 for $|x|>1$, and v_o is the estimated center of the distribution, which is calculated from S_n by taking the two central k_v samples and obtaining a phase-contrast velocity estimate, e.g. $v_o = v_{FOV}/2\pi\ \arg(S_0\, S_1{}^*)$.

No artifacts are expected when the range of velocities is less than v_{FOV}/ρ. In the case of flow jets, aliasing artifacts and blurring at the borders of the distribution may be experienced. However, these distortions are likely to be insignificant, as wider velocity distributions typically have low energy at higher k_v frequencies.

Methods: To determine an appropriate value of ρ for k_v-FOCUSING, we performed simulations using numerical models of the velocity distribution in plug flow and in stenotic and regurgitant flow jets. The method was experimentally validated using MR Doppler in a flow phantom with a wide distribution of velocities, and using spiral FVE to examine aortic valve flow in a healthy volunteer.

Results: Numerical simulations suggest that $\rho = 4$ may be used to image plug flow and flow jets, with no significant artifacts (not shown). In the *in vivo* validation (Fig. 1), the velocity resolution in spiral FVE was improved from 86 cm/s to 33 cm/s using variable-density sampling (160% improvement compared to uniform sampling). With k_v-FOCUSING, velocity resolution was preserved, and aliasing artifacts were eliminated. In Fig. 2, k_v-FOCUSING is applied to MR Doppler in combination with homodyne reconstruction. The method performed well, even though the flow distribution covered a large portion of the v_{FOV}.

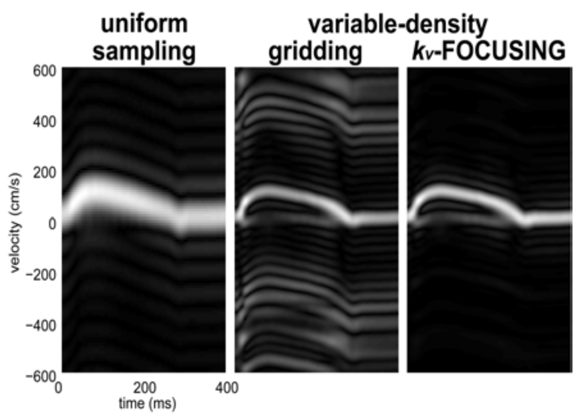

Fig. 1: *In vivo* comparison in spiral FVE [6,7]: healthy volunteer, aortic valve (v_{FOV} = 1200~400 cm/s, 14 k_v samples).

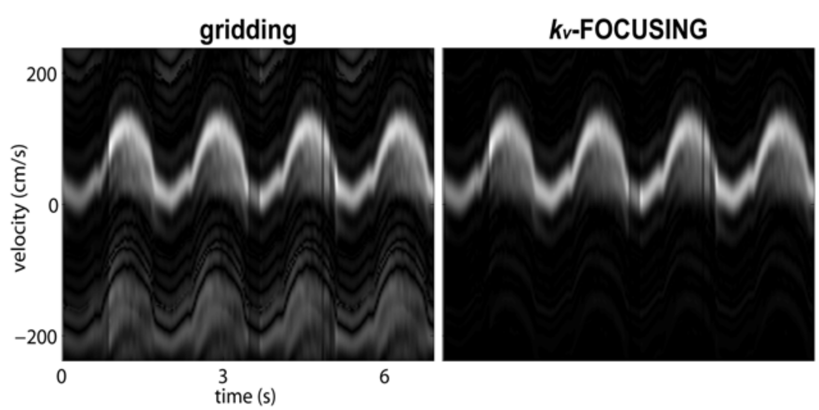

Fig. 2 : Flow phantom imaged with variable-density MR Doppler [3]. (v_{FOV} = 476~173 cm/s, v_{res} = 25 cm/s)

Conclusions: Variable-density sampling along the velocity dimension was used to improve the v_{res} in FVE by up to 160%. Artifacts were eliminated using a novel reconstruction scheme (k_v-FOCUSING), which was validated in numerical simulations, a flow phantom, and *in vivo*, using both spiral FVE and MR Doppler. k_v-FOCUSING can potentially be used with any FVE method, and can be combined with partial Fourier to achieve even higher acceleration factors. Validation studies in patients with flow jets are planned.

References: [1] Moran PR. MRI 1:197, 1982. [2] Tang C, et al. JMRI 3:377, 1993. [3] DiCarlo JC, et al. MRM 54:645, 2005. [4] Jackson JI, et al. IEEE TMI 10:473, 1991. [5] Maeda A, et al. IEEE TMI 7:26, 1988. [6] Carvalho JLA, et al. MRM (in press). [7] Carvalho JLA, et al. ISMRM 2006 #1906. [8] Cukur T, et al. ISMRM 2007 (submitted).