Improving computation of cardiovascular relative pressure fields from velocity MRI

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Introduction

Several studies have been published in which relative pressure fields in the cardiovascular system were computed from velocity MRI using the pressure Poisson equation (PPE). The PPE is an elliptic partial differential equation with Neumann boundary conditions, prescribing specific values for the directional derivatives of the pressure at the boundary of the domain. For the cardiovascular system, the computational domain can be shaped far from rectangular. While there is efficient off-the-shelf code available for the case of rectangular domains, it is considerably more difficult to implement a fast solver on general domains. Therefore a popular workaround has been to use a quasi-rectangular domain by solving the problem iteratively on the rectangular bounding box of the domain and repeatedly modifying the pressure gradients outside the domain to match the current pressure solution. Other methods are available, but are difficult to implement and have not been tested for medical application. Here, we present a very fast multigrid-based solver for the PPE, working directly on the specified domain. Using our solver, we show that the iterative approach with quasi-rectangular domain, depending on the resolution and the geometry of the domain, can lead to significant systematic underestimation of the pressure differences.

Methods

We applied the different methods on several synthetic and in-vitro phantoms as well as on a normal thoracic aorta. For the in-vivo study, a three-dimensional three-directional velocity field in a 278×278×54 mm volume with a spatial resolution of 2.9×2.9×3.0 mm (TR=6.1 ms, TE=3.5 ms, VENC=1.5 m/s, flip angle=8º, NEX=1) was acquired using a 1.5 T MRI scanner (Philips Achieva, Philips Medical System, Best, The Netherlands). Navigator gating was used to minimize respiratory artifacts. The data was corrected on the scanner for the effect of concomitant gradient fields. Correction for eddy current effects was achieved by subtracting a 3D polynomial function, derived from a least-squares fit to stationary tissue, for each velocity component. The computational domain was segmented by thresholding the MRI magnitude signal multiplied with the speed in late systole.

Pressure gradients were computed from the velocity data using central differences within the domain and upwind differences at the edge. From the pressure gradients the relative pressure field was computed by solving the PPE with a variation of the full multigrid method [5], which achieves fast convergence by repeatedly switching back and forth between coarser and finer scales. For irregularly shaped computational domains, explicit rediscretization of the Poisson operator at coarser scales is difficult. This is instead done implicitly using Galerkin coarsening [5].

For comparison, the relative pressure field was also calculated using a rectangular domain and a quasi-rectangular domain. The average pressure within the segmented lumen was adjusted to zero for every computed pressure field to facilitate comparisons.

Result

For the synthetic phantom studies, the different methods resulted in almost the same relative pressure fields for rectangular and convex domains. Using the methods on a c-shaped non-convex domain, a systematic underestimation could be seen using a rectangular and quasi-rectangular domain, while the solution using a non-rectangular domain almost was identical to the mathematical result. This underestimation depended on the resolution of the data, with larger errors for lower resolutions.

For the in-vivo data set, the difference between the original pressure gradients and the pressure gradients computed from the solutions using the different methods was smallest for the proposed method (Figure 1). This is an objective measure of the quality of the different solutions. For this aorta study, an error in the pressure differences using the quasi-rectangular domain of around 30 % could be seen, while the solution using a rectangular domain even differed by an order of magnitude from the solution using the non-rectangular domain.

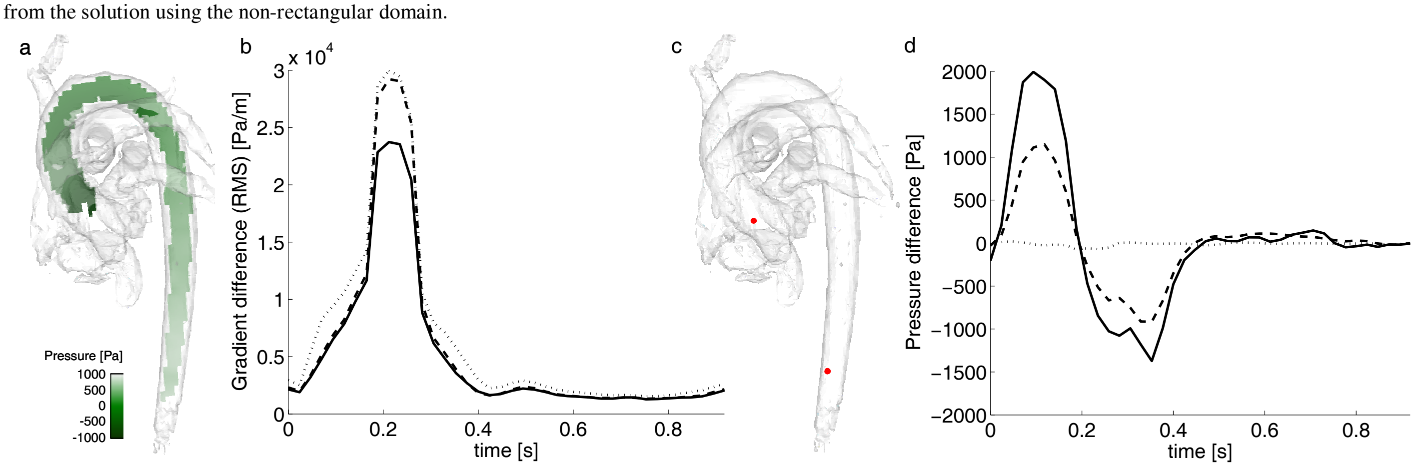

Figure 1. The relative pressure field at late systole (a), the root mean square difference between the original gradients and the gradients of the estimated field (b), and the pressure difference between the ascending aorta and descending aorta (red dots in c)(d) over a cardiac cycle obtained using a rectangular (dotted), quasi-rectangular (dashed), and non-rectangular (solid) domain.

Discussion

We demonstrated that the estimation of relative pressure fields based on velocity MRI using a quasi-rectangular domain can result in substantial errors. For mostly convex domains, such as the heart, the deviations may be small, but for shapes like the aorta the usage of this method can result in a significant underestimation of the obtained relative pressures. Therefore, we recommend always solving the PPE with proper handling of the boundary conditions. Using the multigrid approach, which has been thoroughly validated within other research areas, this can be done extremely fast. Our multigrid-based PPE solver with Galerkin coarsening, working directly on the specified domain, showed in this study to give much better estimates of the relative pressure fields from velocity MRI data compared to commonly used methods.

References

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