# Accelerated Iterative Reconstruction of Temporally Regularized Dynamic MRI

**K. A. Khalsa[1], and J. A. Fessler[1,2]**

[1]Biomedical Engineering, University of Michigan, Ann Arbor, MI, United States, [2]Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI, United States

## Introduction:

The fundamental challenge in dynamic MR imaging is the tradeoff between spatial resolution and temporal resolution. Most traditional dynamic image acquisition methods and associated image reconstruction methods have been based on k-space operations. One acquires a temporal sequence of incomplete k-space sample sets, along with one (or two) complete reference datasets, and then employs some form of data sharing between the sets to find the “missing” data. Finally an inverse FFT is applied to the imputed k-space datasets to form the dynamic image sequence. The Keyhole method and reduced-encoding imaging with generalized-series reconstruction (RIGR) are two examples of data sharing techniques [1,2].

Both Keyhole and RIGR are based on the implicit assumption that the object varies smoothly over time. In this work, we use a model-based reconstruction method that does not require recovering data in the frequency domain, but rather relies on explicit temporal interpolation in the image domain. Similar formulations have been studied in electrocardiography [3-5], and applied to simulated cardiac MRI data [6]. This method requires minimization via the iterative Conjugate Gradient (CG) algorithm, which requires more computation than traditional keyhole methods. To reduce computation, we present an accelerated CG algorithm for dynamic imaging based on Toeplitz matrices and FFT operations [7].

## Theory:

In a dynamic acquisition, the measured data is a collection of $M$ scans, $\mathbf{y}_1,\ldots,\mathbf{y}_M$. We parameterize the dynamic object $f(\vec{r},t)$ during the m-th scan using

$$f(\vec{r},t_m)=\sum_{j=1}^{N} x_{mj}\, p(\vec{r}-\vec{r}_j).$$

Based on the signal equation, the measurement model becomes

$$\mathrm{E}[y_m]=A_m x_m,\quad [A_m]_{ij}=P(\vec{\upsilon}_{mi})\exp(-i2\pi\vec{\upsilon}_{mi}\cdot\vec{r}_j)$$

where $P(\vec{\upsilon})$ is the Fourier Transform of $p(\vec{r})$ and $x_m=(x_{m1},\ldots,x_{m,N})$.

Our reconstruction is formulated as

$$\hat{x}=\arg\min_x \Psi(x)$$

$$\Psi(x)=\sum_{m=1}^{M}\frac{1}{2}\left\|y_m-A_m x_m\right\|^2+\alpha R_1(x)+\beta R_2(x)$$

where $R_1(x)$ and $R_2(x)$ are temporal and spatial regularization terms, respectively. A key step (and the most computationally expensive one) in minimizing $\Psi(x)$ via CG is computing its gradient:

$$\nabla\Psi(x)=-A'(y-Ax)+\alpha\nabla R_1(x)+\beta\nabla R_2(x).$$

We can rewrite this as

$$\nabla\Psi(x)=Tx-b+\alpha\nabla R_1(x)+\beta\nabla R_2(x),$$

where $T = A'A$ and $b = A'y$. Because $b$ does not depend on $x$, it needs to be computed only once. For equally spaced basis functions, $T$ is block Toeplitz with Toeplitz blocks. Calculating the gradient involves multiplying $T$ by the current guess of $x$, which can be done efficiently by embedding $T$ into a block circulant matrix and doing an FFT [7]. This acceleration technique has been investigated previously in field-corrected MR image reconstruction [8], but has not yet been applied to dynamic MRI.

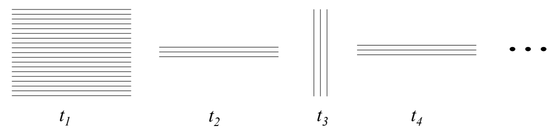


**Fig. 1**: k-space trajectory

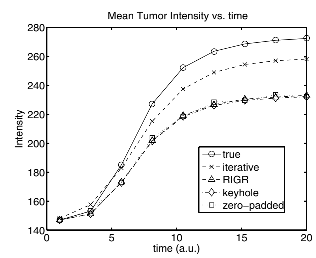


**Fig. 2**: Enhancement curve for simulated lesion

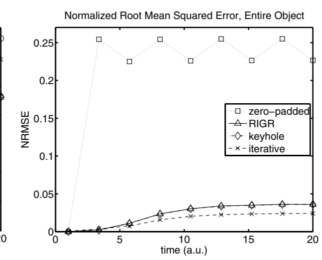


**Fig. 3**: Reconstruction error

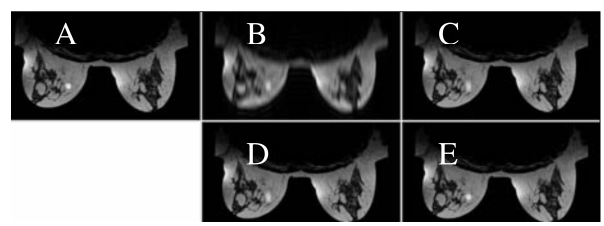


**Fig. 4**: True image (A) and reconstructions with zero-padding (B), Keyhole (C), RIGR (D), and accelerated iterative method (E).

## Results and Discussion:

Contrast agent uptake was simulated using a real bilateral breast image with an inserted (simulated) circular lesion. The lesion exhibited enhancement over time according to the curve in Fig. 2, while the rest of the image remained static. We generated k-space data for one reference frame (prior to enhancement) and eight subsequent, undersampled (by a factor of 16) frames, using the k-space trajectory in Fig. 1. The data was reconstructed using four methods: zero-padding, Keyhole, RIGR, and our accelerated iterative temporally regularized method. Fig. 3 shows the normalized root mean square (NRMS) error for each of the reconstruction methods. The Fig. 4 shows reconstructed images from the 5th timepoint. The regularized method yields lower NRMS error and reduced blur. For this study, the Toeplitz-modified CG method was 1.7 times faster than the original CG method. In summary, iterative reconstruction using temporal regularization shows promise for dynamic contrast enhanced MRI, both in terms of spatial resolution and temporal resolution. The computation time for this method can be greatly reduced by using the proposed Toeplitz acceleration of the CG algorithm.

**Acknowledgments:**

This material is based upon work supported by a National Science Foundation Graduate Research Fellowship.

**References:**

[1] van Vaals JJ, et al. JMI 1993; 3(4): 671-5.
[2] Liang Z-P, et al. IEEE TMI 2003; 22(8): 1026-30.
[3] Oster HS, et al. IEEE BME 1992; 39(1): 65-75.
[4] Brooks DH, et al. IEEE BME 1999; 46(1): 3-18.
[5] Greensite F. IEEE BME 2003; 50(10): 1152-9.
[6] Samsonov A. ISMRM 2005; 2309.
[7] Chan RH, et al. SIAM Review 1996; 38(3):427-82.
[8] Fessler JA, et al. IEEE TSP 2005; 53(9): 3393-402.