Criterion to Accelerate Time-Resolved MRI Based on the Corners of a 4D k-Space

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Introduction: The spatial resolution contributed by data in the corners of k-space is angular-dependent, and can only be appreciated when image interpolation such as zero-filling is applied (1). Reducing the acquisition time by skipping the acquisition of data in the corners of spatial k-space has become a widely-adopted method. For example, 2D spiral acquisitions and 3D Cartesian acquisition need not fill approximately 21% of the k-space (in *kx-ky* and *ky-kz*, respectively). Non-Cartesian 3D acquisitions like spherical shells (2,3) can save approximately 47% of the imaging time, because the volume of an inscribed sphere is only about half the volume of the circumscribing k-space cube.

Accelerated time-resolved methods such as BRISK(4), TRICKS(5), TREAT(6), keyhole(7), k-t BLAST(8), and HYPR(9) have attracted great interest, particularly for CE-MRA and cardiac applications. The purpose of this work is to provide a criterion to define the corners of a joint spatial-temporal k-space. It is shown that when a total of four dimensions of corners (three spatial and one temporal) is not acquired, the time-savings is approximately 70%. The same method can be used to greatly accelerate the acquisition of other types of joint k-spaces, such as those obtained with six-dimensional spatial-velocity Fourier encoding.

Theory and Methods: Consider a joint 4D k-space with three spatial k_x, k_y, k_z, and one temporal coordinate k_t. The extent of each k-space coordinate is related to the resolution along that direction, e.g., $-k_{0X} \le k_x \le k_{0X}$ with spatial resolution $\Delta x = 1/(2k_{0X})$. The temporal k-space coordinate is the sampling frequency *f* of the time series, so that $k_t = f$, and by analogy its extent is $k_{0T} = 1/(2\Delta t)$, where Δt is the temporal resolution. The corners of this joint k-space lie inside the 4-dimensional rectangular prism

$$-k_{0X} \le k_x \le k_{0X}\ ,\ -k_{0Y} \le k_y \le k_{0Y}\ ,\ -k_{0Z} \le k_z \le k_{0Z}\ ,\ -k_{0T} \le k_t \le k_{0T} \qquad [1]$$

while lying outside the 4-dimensional ellipsoid:

$$\left(\frac{k_x}{k_{0X}}\right)^2 + \left(\frac{k_y}{k_{0Y}}\right)^2 + \left(\frac{k_z}{k_{0Z}}\right)^2 + \left(\frac{k_t}{k_{0T}}\right)^2 = 1 \qquad [2]$$

Results: The sampling frequency *f* is typically expressed over a range of positive values $0 \le f_+ \le 2k_{0T} = 1/\Delta t$; re-arranging Eq. [2] yields the frequency extent of the 4D-ellipsoid:

$$f_+ = \frac{1}{\Delta t}\sqrt{1 - \left(\frac{k_x}{k_{0X}}\right)^2 - \left(\frac{k_y}{k_{0Y}}\right)^2 - \left(\frac{k_z}{k_{0Z}}\right)^2} \qquad [3]$$

The content (also called "hyper-volume", not to be confused with HYPR of Ref. 9) of the 4D-rectangular prism defined in Eq. [1] is simply the product of the lengths of its four total extents: $16 \times k_{0X} \times k_{0Y} \times k_{0Z} \times k_{0T}$. The hyper-volume (10) of an ellipsoid as in Eq. [2] of dimension *d* is given in Table 1, where $\Gamma(x) = (x-1)!$, and $\Gamma(0.5) = \pi^{1/2}$. From Table 1, it can be seen that as the dimensionality *d* increases, the relative content in the corners of k-space increases rapidly. For example, if *d* = 7 (e.g., 3 spatial, 3 velocity-encoded, and 1 temporal axis) then 96.3% of the content lies in the corners, offering a 25-fold acceleration. This rapid convergence towards 100% can be verified by applying Stirling's approximation for the gamma function.

Dimension *d*	Content of inscribed hyper-ellipsoid	Content in corners (% of content of hyper-rectangle)
1	$2k_{0X}$	0
2	$\pi k_{0X} k_{0Y}$	21.5
3	$\frac{4\pi}{3} k_{0X} k_{0Y} k_{0Z}$	47.6
4	$\frac{1}{2}\pi^2 k_{0X} k_{0Y} k_{0Z} k_{0T}$	69.2
d	$\frac{2}{d}\frac{\pi^{d/2} k_{0X} k_{0Y} \ldots k_{0d}}{\Gamma(d/2)}$	$100\times\left(1 - \frac{\pi^{d/2}}{d\ 2^{d-1}\Gamma(d/2)}\right)$

Discussion: Equation [3] provides a quantitative criterion to specify which time-frames are most important to acquire in various regions of the spatial k-space. According to Eq. [3], the center of the spatial k-space is acquired at full temporal resolution $f_+ = 2k_{0T}$, while the periphery need only be acquired at a reduced rate. This result is consistent with methods such as BRISK, TRICKS and TREAT (see, for example, the data presented in Fig. 1 of Ref. 4). The proposed method, however, specifies how to accelerate the acquisition quantitatively, and is model-independent. Temporal resolution requirements may dictate that data are sampled less frequently than suggested by Eq. [3], but that equation provides a criterion to determine when loss of spatial-temporal resolution can be expected, i.e., whenever data inside the hyper-ellipsoid are not acquired (unless special methods like partial Fourier reconstruction or parallel imaging can be employed).

Although the criterion of Eq. [3] is quite general, it does assume that the spatial-temporal FOV roughly matches the properties of the object. For example, consider a simple 2D case, where data in the corner of the *kx-ky* space are often apodized, or not acquired at all. The robustness of that method has been validated through years of clinical use, yet it could in principle produce poor results if the 2D FOV is selected to be much too large, so that the object (e.g., the entire head) becomes a point source and the raw data power is no longer concentrated in the center of k-space. By analogy, the validity of the proposed method relies on avoiding unreasonable selections for the number of time frames and the temporal resolution Δt .

The proposed method should be particularly useful for time-resolved spherical shells acquisitions (2-3), because of their true 3D-centric nature. Finally, we mention that the proposed method does not require phased-array coils, so it is compatible with other acceleration methods such as parallel imaging for even greater acceleration.

References: 1. Bernstein MA, et al J Magn Reson Imaging 2001;14: 270-80. **2**. Shu Y, et al, Magn Reson Med 2006; 56: 553-562. **3**. Shu Y, et al, Magn Reson Imaging 2006; 24: 739-49. **4**. Doyle M, et al., Magn Reson Med. 1995; 33: 163-70. **5**. Korosec FR, et al., Magn Reson Med. 1996; 36: 345-51. **6**. Pinto C, et al., Vasc Interv Radiol. 2006; 17: 1003-9. **7**. van Vaals JJ, et al., J Magn Reson Imaging 1993; 3: 671-5 **8**. Tsao J, et al., Magn Reson Med. 2003; 50: 1031-42. **9**. Mistretta CA, et al., Magn Reson Med. 2006; 55: 30-40. **10**. Cairns SS, Bull. Am. Math Soc. 1959, 65: 327-328.