Block-wise FFT: a Memory Efficient FFT technique for Magnetic Resonance Imaging

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Introduction

Fast Fourier Transform (FFT) is a crucial reconstruction step in Magnetic Resonance Imaging (MRI). With the increase of the spatial/temporal resolution of MRI, FFT may suffer from memory limitation. For example, after gridding, the *k*-space data could be of size 1024 pixel × 1024 pixel × 1024 pixel for acquisition of a trajectory with 1024 readouts per VIPR projection, which requires at least 16 GBytes if data are stored as double precision complex numbers. Most computers today cannot store such a large matrix in memory, not to mention performing FFT to reconstruct images. Some reconstruction techniques for non-rectilinearly sampled data, such as iterative next neighbor regridding (INNG) [1], require large rescaled matrices to avoid the difficulty in optimizing the density compensation function. Consequently, they suffer from extensive memory cost, especially when dealing with data with three or more dimensions. Here, a new algorithm based on fractional FFT (fFFT) is proposed, in which FFT is performed block by block to obtain the entire transformed matrix in the frequency domain. This method is memory efficient as it is unnecessary to concurrently hold all data in memory for calculation; hence, it can be applied to reconstruction with very large matrix size.

Methods

Based on chirp *z*-transform, the fractional Fourier transform (fFT) [2] is a generalization of the classical discrete Fourier transform (dFT). fFT is defined as $G_k(x,\alpha)=\sum_{j=0}^{m-1} x_j e^{-2\pi i jk\alpha}$, where α is an arbitrary number, and m is the length of the signal. Ordinary dFT is a special case of fFT with $\alpha=1/m$. Similarly, the inverse of dFT is also a special case of fractional inverse Fourier transform (fIFT). Fractional fast Fourier transform (fFFT) provides an efficient method to perform fFT using ordinary FFT.

With α determined by the ratio of the block size to the original matrix size, fFFT can be used for FFT calculations using only part of the original data. As shown in Fig. 1, in our block-wise FFT (BwFFT) method, a block of the original data (in the real space) is loaded and its contribution to every block in the resulted matrix (in the *k*-space) is calculated using fFFT assuming zero padding. Appropriate phase correction is applied to the resulted matrix to account for the location of the original data block according to the Shifting Theorem. The final transformed matrix is obtained by summation over the matrices resulted from all original data blocks.

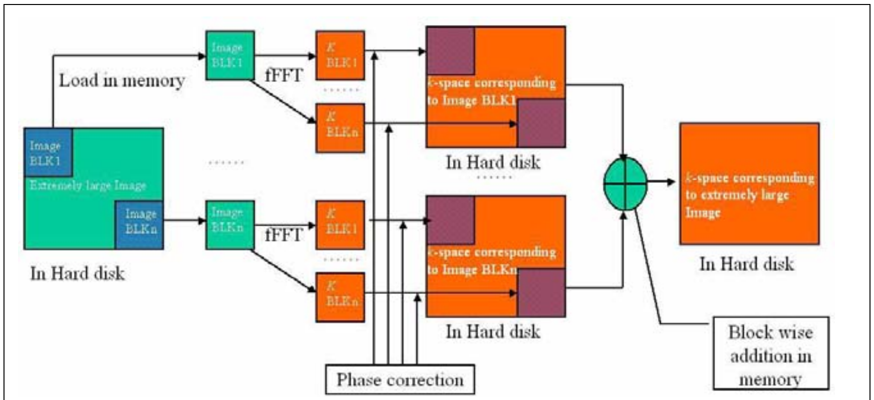

Figure 1. The flowchart for the block wise FFT

Implementation and Application

Both theoretical deduction [3] and our experiments showed that the results from our BwFFT and traditional FFT are identical. Therefore, the BwFFT method can be used when memory is insufficient to carry out traditional FFT. We also employed the BwFFT method in the INNG for reconstruction from non-uniformly sampled, three-dimensional k-space data, where the original INNG algorithm could not handle the calculation due to memory limitation. Using our method, as shown in Fig. 2, the entire rescaled matrix was never formed in memory. Each time, only a block of this matrix was employed. Furthermore, only the center part of the resulted inverse Fourier transform (IFT) matrix of this virtual rescaled matrix is calculated using block-wise inverse FFT (IFFT), which is the location for the reconstructed image. Comparing with the flow chart of the basic INNG algorithm (in Fig. 1 of reference [1]), our approach greatly reduces the memory requirement from 3×sN×sN×sN to 2N×N×N, with s = 4 or 8 in many cases. It is also important to note that our approach reduces necessary computation while reaching the same quality for reconstructed images.

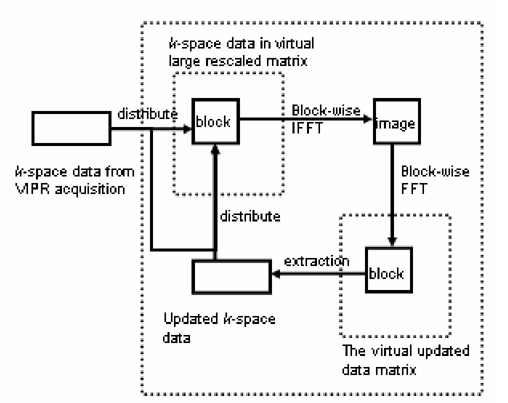

Figure 2. block-wise FFT for INNG algorithm with less memory requirements

Discussion and Conclusion

Memory limitation may be a bottle-neck in reconstructing MRI images using FFT. The block-wise FFT technique described here provides a memory efficient approach to perform FFT for matrices with extremely large size without compromising the accuracy. It can be applied to reconstruction of MRI images with large matrix size, especially for MRI with non-Cartesian trajectory in a multi-dimensional space.

Reference:

[1] Moriguchi H, et.al., MRM 2004;51:343-352.
[2] Bailey DH, et.al., SIAM Review 1991;33:389-404