An iterative off-resonance and signal decay correction for improved R_2^* mapping

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Introduction: The use of echo planar imaging (EPI) trajectories has been proposed to accelerate k-space sampling in a multiple-echo R_2^* mapping experiment [1]. In this technique, an EPI echo train is used to acquire several k-space lines for each echo time. The EPI echo trains follow each other until the desired number of echoes has been acquired. This multiple-echo EPI technique is therefore associated with an increased readout time. Hence, images reconstructed for each echo time with a standard Fourier transform suffer from off-resonance and relaxation artefacts, which eventually corrupts the estimation of the relaxation rate map [2]. This work proposes a new, iterative reconstruction algorithm to correct these two effects and to compute an artefact-free relaxation rate map. Gradient-echo images, off-resonance and relaxation rate maps are jointly estimated on the basis of the multiple-echo EPI data solely.

Theory: The input data of the proposed joint estimation algorithm consist solely of the samples $s_{l,k}$ measured at echo times TE_l and k-space nodes k_k of a multiple-echo EPI R_2^* mapping experiment. A complex map z can be defined as $z = R_2^* + i\omega$, with R_2^* the relaxation rate map and ω the off-resonance map. In the following, a uniform approach is taken to treat simultaneously off-resonance and relaxation-induced signal decay. The algorithm starts with a null map. Each iteration of the reconstruction algorithm consists of three steps. First, the values of the transverse magnetization $m_{l,\rho}$ are computed by solving the following system of linear equations:

$$s_{l,\kappa} = \sum_{\rho} m_{l,\rho} e^{-t_{\kappa} z_{\rho}} e^{ik_{\kappa} r_{\rho}} \quad , \tag{1}$$

with the conjugated gradient (CG) algorithm, given the current estimate of the map *z*. The CG-algorithm proceeds by applying direct and adjoined evaluations of the sum in Eq. 1, which is a computationally demanding matrix-vector multiplication. To accelerate these evaluations, it is proposed to generalize the gridding approach introduced in Ref. [3] for the correction of off-resonance in order to obtain a uniform treatment of off-resonance and relaxation. Hence, the exponential term $\exp(-t_k z_p)$ is approximated by a finite sum of exponentials, which enables the application of a, possibly non-equispaced, Fast

Fourier Transform algorithm [4].

In the second step, the complex magnetization m_{ρ} at echo time zero, and the complex map z_{ρ} are updated on the basis of the time series $m_{l,\rho}$ according to the following relation:

$$m_{l,\rho} = m_{\rho} e^{-TE_l z_{\rho}} .$$

Eq. (2) is solved by means of a least squares algorithm; the solution of this non-linear optimization problem is found using e.g. Powell's method [5]. The pair (m_{ρ_c}, z_{ρ}) can be estimated independently for each voxel ρ , so that this calculation can be highly parallelized. In the last step, the quadratic norm r^2 of the residuals, computed with the new values of the magnetization m_{ρ} and the complex map z_{ρ_c} is evaluated, according to:

$$r^{2} = \sum_{l,\kappa} |s_{l,\kappa} - \sum_{\rho} m_{\rho} e^{-(TE_{l} + t_{\kappa})z_{\rho}} e^{ik_{\kappa}r_{\rho}} |^{2}, \qquad (3)$$

These three steps are repeated, until the minimization of r^2 is achieved. Interestingly, the proposed algorithm is a generalization of the standard estimation method: the first iteration yields the R_2^* map estimate obtained when no off-resonance and relaxation correction is applied, and the following iterations gradually improve the estimate by correcting the images at the consecutive echoes with refined values of the off-resonance and the relaxation rate.

Methods: The proposed reconstruction algorithm was evaluated with simulated data of a multiple-echo EPI experiment. A slightly modified Shepp-Logan Phantom of size 256x256 was chosen as image of the transverse magnetization. The field map was parabolic in the range [0Hz, 125Hz]. The relaxation map, simulated on the basis of the Shepp-Logan Phantom, had values in the range $[5s^{-1}, 50s^{-1}]$. Echo planar trajectories with EPI factors ranging from 2 to 8 were simulated. The echo spacing between echo images increased accordingly from 4 to 16ms, while a constant repetition time was assumed. With increasing echo spacing, the estimation of the off-resonance map becomes prone to wrapping errors. To cope with this difficulty, a second multiple-echo time series was simulated, with the same echo spacing, but with the first echo time shifted by 1ms. The non-equispaced echoes are then processed by means of the reconstruction algorithm as described above. Eventually, the overall acceleration factor in comparison to a standard Cartesian multi-echo experiment ranged from 1 to 4.

Results: The evolution of the estimation error as a function of the iteration number is shown in Fig. 1 for an EPI factor of 8. Convergence was reached after 4 iterations. The corresponding estimates of the magnitude image, the off-resonance map, and the R_2^* map are shown in Fig. 2. As can be seen on the R_2^* difference map, the initial off-resonance artefacts, causing geometric distortions, are removed by the proposed correction.



Fig. 1: Normalized RMS Error of image, off-resonance and R_2^* estimates as a function of iteration number, for an EPI factor of 8.



Fig. 2: Magnitude image (a), off resonance map (b), and R_2^* map (c) after convergence, for an EPI factor of 8. The difference between final and initial estimates of the R_2^* map is shown in (d).

Discussion and conclusion: By correcting for off-resonance and relaxation artefacts, the proposed iterative algorithm improves the estimation of R_2^* with fast imaging trajectories having long readout times. Off-resonance, relaxation map, and gradient-echo images are jointly estimated in a unified framework, so that no pre-measurement is required. Applying a generalized gridding algorithm to approximate the exponential term in the CG algorithm accelerated the computations substantially.

References:

[1] Speck, MRM, 40:243-248 (1998) [2] Olafsson et al, ISMRM, #45, (2004). [3] Eggers et al, ISMRM, #2969, (2006). [4] Dutt et al, SIAM J Sci Stat Comput, 14:1368-1393 (1993). [5] Press et al, Numerical Recipes in C (1992).