Improving Super-resolution by Adopting Phase-scrambling Fourier Imaging

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INTRODUCTION This paper presents a novel resolution improvement technique for the echo signal obtained in the phase-scrambling Fourier imaging technique (PSFT) [1]. Resolution improvement is performed using the super-resolution technique based on Gerchberg's algorithm [2]. Because of the existence of quadratic phase modulation used in the PSFT, spatial resolution is fairly improved compared to the case of applying the signal obtained in the standard Fourier imaging technique.

METHODS Phase-Scrambling Fourier Transform (PSFT) imaging is a technique whereby a quadratic field gradient is added to the pulse sequence of conventional FT imaging in synchronization with the field gradient for phase encoding. The signal obtained in PSFT is given by Eq. (1) [1]:

$$v(k_x,k_y)=\int\int_{-k_m/2}^{k_m/2}\rho(x,y)e^{-j\gamma b\tau\left(x^2+y^2\right)}e^{-j\left(k_x x+k_y y\right)}dxdy=\mathrm{rect}\left(\frac{k_x}{k_m}\right)\cdot F\left[\rho(x,y)e^{-j\gamma b\tau\left(x^2+y^2\right)}\right], \tag{1}$$

where $\rho(x,y)$ represents the spin density distribution in the subject, γ is the magnetogyric ratio, b and τ are the coefficient and impressing time of the quadratic field gradient, respectively, and rect(k_x/k_m) is the top-hat function to introduce the effect of limited sampling length k_m. Figure 1 shows the iteration procedure using Gerchberg's super-resolution algorithm [2] in the case of a one-dimensional signal. Figure 1(a) is an obtained echo signal sampled by N data points at intervals of Δk_x, and Figure 1(b) shows the inverse Fourier transform of the signal of Figure 1(a). The reconstructed image can be obtained by canceling the quadratic phase modulation by applying exp[jγbτx^2]. The reconstructed image of Figure 1(c) is written as

$$\rho_{rec}(x)=\exp\left[j\gamma b\tau x^2\right]\cdot F^{-1}\left[\mathrm{rect}\left(\frac{k_x}{k_m}\right)\cdot v(k_x)\right]=\exp\left[j\gamma b\tau x^2\right]\cdot\left\{\mathrm{sinc}(k_m x)*\rho(x)\exp\left[-j\gamma b\tau x^2\right]\right\}. \tag{2}$$

where * denotes the convolution integral. Figures 1(c) through 1(d) show the procedure used to estimate the echo signal using the reconstructed image of Figure 1(c). Here, the real part of the image shown in Figure 1(c) is taken as a constraint such that the subject function is a real value:

$$\mathrm{Re}\left[\rho_{rec}(x)\right]=\frac{1}{2}\exp\left[j\gamma b\tau x^2\right]\cdot\left[\mathrm{sinc}(k_m x)*\rho(x)\exp\left[-j\gamma b\tau x^2\right]\right]+\frac{1}{2}\left[-j\gamma b\tau x^2\right]\cdot\left[\mathrm{sinc}(k_m x)*\rho(x)\exp\left[j\gamma b\tau x^2\right]\right] \quad . \tag{3}$$

The calculated PSFT echo signal $v_{est}(k_x)$ is then written as follows:

$$v_{est}(k_x)=F\left[\mathrm{Re}\left[\rho_{rec}(x)\right]\exp\left[-j\gamma b\tau x^2\right]\right]=\frac{1}{2}\mathrm{rect}\left(\frac{k_x}{k_m}\right)\cdot F\left[\rho(x,y)e^{-j\gamma b\tau\left(x^2+y^2\right)}\right]+\frac{1}{2}\mathrm{rect}\left(\frac{k_x}{k_m}\right)\cdot F\left[\rho(x,y)e^{j\gamma b\tau\left(x^2+y^2\right)}\right]*F\left[e^{-j2\gamma b\tau\left(x^2+y^2\right)}\right] \quad . \tag{4}$$

The first term of the right-hand side of the Eq. (4) is half as great as the obtained echo signal of Eq. (1) and does not have a frequency component over the band limit $k_m/2$. However, the second term has a frequency component beyond the band limit $k_m/2$, because the band-limited signal is convolved with the function F[exp[-j2γbτx^2]]. When the top-hat function rect(k_x/k_m) can be ignored, the second term becomes the target PSFT signal F[ρ(x)exp[-jγbτx^2]], so that the iteration from Figure 1(b) to Figure 1(f) gradually restores the frequency spectrum beyond the band-limit k_m, as shown in Figure1(e) which results in the improvement of spatial resolution. The existence of the convolution of F[exp[-j2γbτx^2]] in Eq. (4) is different than the standard Gerchberg's super-resolution algorithm and is important in restoring the signal spectrum.

EXPERIMENTS Figure 2 shows the results of simulation experiments. The noise-contaminated signal consists of a 128× 128 data matrix. After 10 iterations, the resolution was improved by 1.3 times by using the proposed algorithm. Almost the same improvement in resolution was attained using the data obtained experimentally.

CONCLUSION A new resolution improvement technique using the signal obtained by the phase-scrambling Fourier imaging technique is presented and demonstrated. Super-resolution was applied to improve the spatial-resolution. The resolution was shown to be improved by 1.3 times compared to the image obtained by the standard Fourier Imaging technique.

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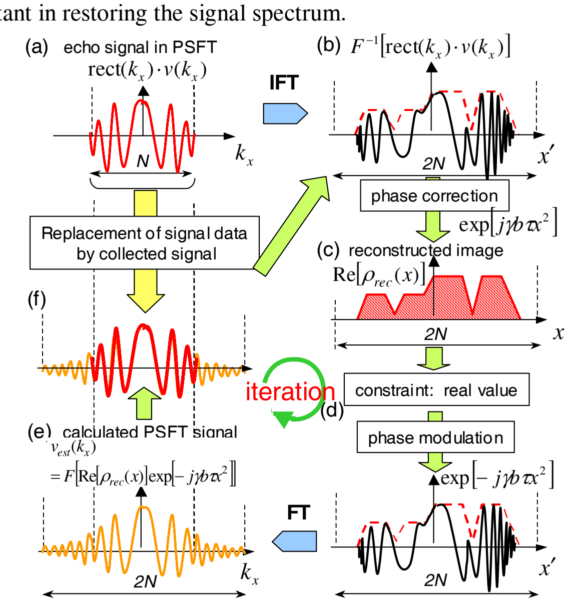

Fig.1 An algorithm of super-resolution using PSFT signal

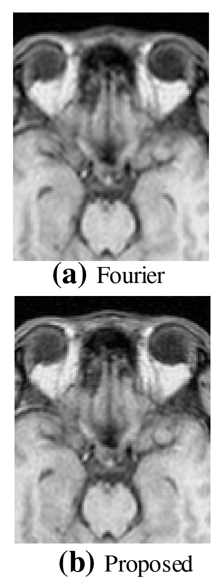

Fig.2 Comparison of images