Maximum Likelihood estimation of diffusion parameters with a Rician noise model

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Introduction

When estimating the parameters of the diffusion tensor it is common to choose those that minimise the sum of squared differences between the observed data and the predictions made by the model. This is equivalent to a Maximum Likelihood (ML) estimate given an assumption of normal distributed errors. In reality the errors have a Rician distribution, but when the SNR is sufficiently high they are well approximated by a normal distribution. For poor SNR, which could be caused by scanning with high resolution or by using high *b*-values, it has been shown that diffusion is underestimated [1].

Theory

The likelihood $L(\theta \mid y)$ function allows one to calculate the probability of observing some data y given the parameters θ. The choice of θ which maximises the likelihood is the ML estimate. In this work we derive expressions for the likelihood, its gradient and the Fisher information matrix for a Rician noise model. These are used for rapid and robust ML estimation of the parameters of the diffusion tensor.

The diffusion tensor was modelled in terms of its spectral decomposition, i.e. the parameters were three angles describing the orientation of the tensor and three eigenvalues quantifying the diffusion. By applying non-negativity constraints to the eigenvalues the solution was constrained to the space of valid (positive definite) tensors.

In addition we describe how the Fisher information can be used to design optimal acquisition schemes.

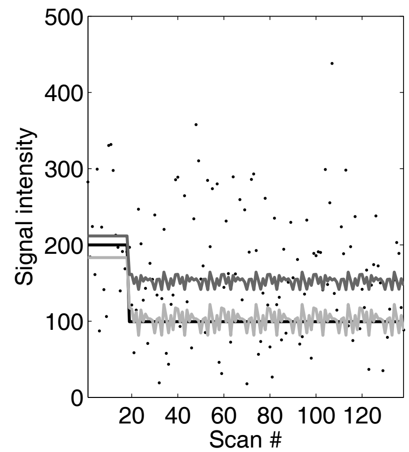

Figure 1 The solid black line shows true signal, the black dots signal with noise, the dark grey the signal predicted by a Gaussian model and the light grey the predictions by a Rician model. Note how the Gaussian predictions run through the centre of the data points while the Rician model is much closer to the true signal.

Materials and Methods

Data with additive Rician noise was simulated for different SNR and/or different b-values. Diffusion parameters were estimated from the simulated data in an ML sense using a Gaussian or a Rician noise model. In addition data was acquired on a Rhesus monkey with 0.8mm cubed resolution, a *b*-value of 1000mm^2/s and a total of 567 acquisitions.

Results

The simulations showed that the estimated parameters were less biased than those obtained for a Gaussian noise model over a wide range of medium to low SNR and/or high b-values. The precision of the Rician estimates was more affected by low SNR than were the Gaussian estimates. Accuracy (which take bias and precision into account) was higher for the Rician estimates for all but the lowest SNR (<1).

Discussion

We have found that it is feasible to estimate diffusion tensor parameters using a Rician noise model. Short execution time and robust convergence was achieved by a combination of Fisher-scoring and a Levenberg-Marquardt minimisation scheme. Whenever quantitative values are the main concern, such as when comparing different bio-physical models of diffusion a Rician noise model should be used. Higher precision Rician estimates can be obtained by (spatial) pooling of the estimate of the variance parameter of the process.

References

[1] Jones & Basser, 2004, MRM 52:979-993.

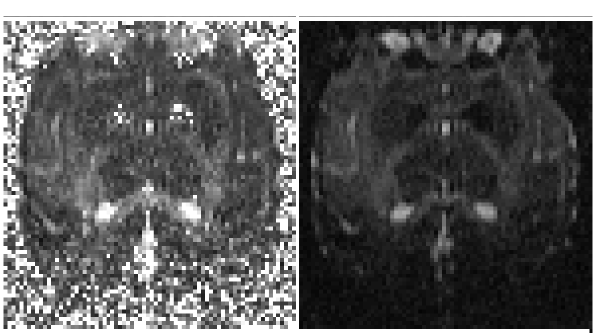

Figure 3 The largest eigenvalue obtained using a Rician (left) and a Gaussian (right) noise model. The estimated diffusion is 30% (white matter) to 100% (CSF) larger for the Rician noise model.

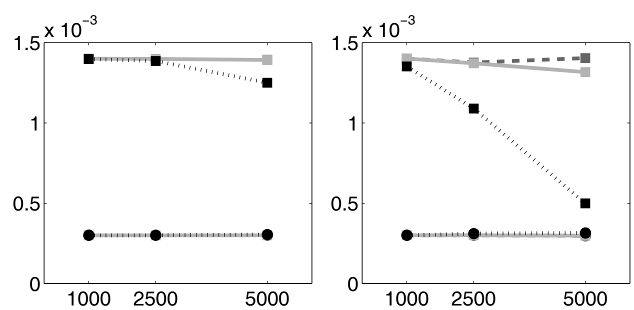

Figure 2 Estimated diffusion along the long (squares) and short (circles) axis of a white matter tensor as a function of *b*-value for data with an SNR of 50 (left) and 10 (right). The estimates obtained with a Gaussian noise model are shown as black dotted lines and for a Rician model as solid and dashed grey lines.