A fractal dimension for exploratory fMRI analysis

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Introduction

Several approaches to fMRI time series analysis without any use of a stimulation paradigm exist, see e.g. [1] for applications of Fuzzy Clustering and Principal Component Analysis or [2] for a Wavelet–Based Multifractal Analysis. We propose a novel method based on a special fractal dimension, the extended counting method xdim [3], which is well suited for data mining due to algorithmic simplicity and short CPU times. An improved robustness of xdim against distortions in comparison with the frequently used box counting method was shown in [4]. The applicability of the method to time series analysis is demonstrated by an analysis of fractional Brownian motion and by an investigation of visually stimulated real fMRI data. The fractal dimensions of Brownian motion [5] are reasonably reproduced for short time series, the fMRI analysis indicates an interesting discrimination between white matter, gray matter and stimulated voxels.

Methods and Materials

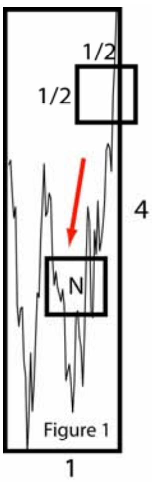
I) For numerical calculation of the fractal dimension any time series is processed in the following way: The amplitude of the time series is scaled to fill a fixed rectangle ((Max[time]-Min[time])/(Max[amplitude]-Min[amplitude])=1/4). The discretization of the time axes determines the gridsize for this rectangle. Discrete amplitudes for neighbouring time steps are then interpolated linearily. Xdim is finally determined by the maximum overlap N between a large moving window (windowsize=(Max[time]-Min[time])/2) and the irregular geometric structure derived from scaling and interpolation, xdim=Log[N] /(Log[windowsize]-Log[gridsize]) [3], see Figure 1. **II)** For validation, fractional Brownian motion of order α with known Hausdorff dimension F= 2-α [5] was simulated for $0<\alpha<1$ over 70 time steps and 10000 replications per α to characterize bias and variance of xdim. **III)** fMRI data were collected on a 1.5 T clinical scanner (multi-slice single-shot gradient-echo echoplanar images, TR=3000 ms, TE=60 ms, flip angle= 90°) during visual stimulation periods. Three on and four off periods (10 images each) were measured from the visual cortex resulting in time-series of 70 images with temporal resolution 3 s, nominal spatial resolution was 2.9x2.9x5 mm. To compensate motion artefacts, images were realigned using AIR algorithm [6], in addition the time series were detrended.

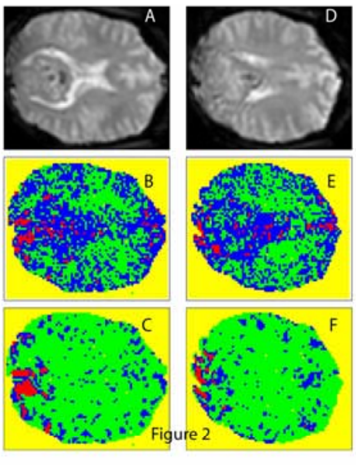
Results

I) In Table 1 the exact Hausdorff dimensions F and expectation values of the numerical approximations xdim are given for different orders α. Good precision is achieved for standard (α=0.5) and nearly standard Brownian motion, else at least a reasonable accuracy is produced. The deviations are due to the shortness of the time series (in agreement with the fMRI data 70 time steps are chosen) and due to inherent limitations in discrete approximations of fractal dimensions [4]. **II)** Results for two empirical slices with visual stimulation are presented in Figure 2. Upper panels give the anatomical patterns (T2*). Middle panels show xdim per voxel (red: xdim<1.665, blue: 1.665<=xdim<1.76, green: >=1.76). Lowest panels give Kendall τ correlations with the boxcar stimulus (red: τ>0.4, blue: 0.2<τ<=0.4, green: 0<=τ<0.2). Xdim detects approximately the same stimulated voxels like τ, in addition it discriminates roughly between gray matter and white matter voxels which show similar irregularity like noisy background voxels (not shown).

Discussion

No a priori assumption about autocorrelation or the noise behaviour of the time series is necessary in this approach, xdim measures the irregularity of a scaled graph. The results for the fractional Brownian motion indicate that our approach is well suited even for analysis of nonstationary time series. Due to the shortness of the time series and finite grid effects a perfect agreement between F and xdim cannot be expected, however a relative comparison between differently irregular time series seems to be feasible. The results for real fMRI data can be interpreted like follows: The time series in white matter show a high irregularity similar to background noise (see Figure 2, xdim: mainly green voxels), as their low frequency modulation caused by the metabolic needs is lower than in gray matter (mainly blue voxels). CSF from ventricles seems to regularize the time series also (mainly blue voxels). Lowest xdim (red voxels) can be found in visually activated voxels, where the BOLD effect due to stimulation dominates, compare B, C and E, F, but also in some voxels close to CSF. Improvements may be achieved by replacing the linear interpolation between the discrete amplitudes by convenient fractal interpolations [5], applications to more data sets are in progress.





F	E[xdim]	σ[xdim]
1.1	1.32	0.12
1.3	1.46	0.08
1.5	1.56	0.06
1.7	1.65	0.05
1.9	1.75	0.03

Table 1: Exact fractal dimensions F versus expectation and standard deviation of xdim for Brownian motion.

Figure 1: Moving window for a graph, N indicates max. overlap.

Figure 2: fMRI analysis for two slices (A,D). B,E show xdim, red (xdim<=1.665), blue (1.665<xdim<1.76), green (xdim>=1.76), C,F show Kendall τ correlation with stimulating boxcar, red (τ>=0.4), blue (0.2<τ<0.4), green (0<τ<=0.2).

References

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