Physical Model of Weiskoff EPI Temporal Stability Test

A. S. Barnett[1], and J. Bodurka[2]

[1]Clinical Brain Disorders Branch, National Institute of Mental Health, Bethesda, MD, United States, [2]Functional MRI Facility, National Institute of Mental Health, Bethesda, MD, United States

Introduction

The Weisskoff test for temporal stability consists of a plot of the reciprocal of the temporal signal-to-noise ratio 1/TSNRn vs side length n for a square ROI defined for a time series of epi images of a uniform phantom. The temporal signal-to-noise ratio is the average divided by the standard deviation of the time series of the sum of the signals in an ROI. Weisskoff (1) observed that the data diverge from the expected 1/n dependence, and suggested that the divergence could serve as a measure of scanner instability. Bodurka (2) later observed that the data are well described by a function of the form

$$1/\mathrm{TSNRn} = \sqrt{(1+(n*1*\mathrm{SNR})^2)}/(n*\mathrm{SNR}) \quad , \qquad [1]$$

where the parameters λ and SNR characterize the scanner performance. The present work presents a derivation of Equation [1] from the simple assumption that the scanner instability causes the gain from time point to point to vary stochastically, and relates the parameter λ to the standard deviation of the gain.

Theory

The signal S_i of voxel i in an MR image of a uniform phantom can be written as $S_i = S_0 + N_i$, where S_0 is the "true" signal and N_i is the noise. If N_i is primarily thermal noise, it is well described as a Gaussian random variable with mean 0 and variance σ_N^2 : $\overline{N_i} = 0$, $\overline{N_i^2} = \sigma_N^2$. We assume that the statistics of the noise is the same in all voxels, and that the noise in different pixels is uncorrelated: $\overline{N_i N_j} = 0$. In the preceding expressions, $\overline{a}$ is the expectation value of the random variable a .

Consider measurements of a time series of images of a uniform phantom. Suppose that rf instability in the scanner causes the gain of each image to be different. The value of pixel i in image k can be written as $S_{i,k} = g_k(S_0 + N_i)$, where the gain g_k is a random variable with mean 1 and variance σ_g^2 : $g_k = 1 + \alpha_k$, $\langle \alpha_k \rangle = 0$, $\langle {\alpha_k}^2 \rangle = \sigma_g^2$, and $\langle \alpha_k \alpha_l \rangle = 0$, α is a random variable that describes the gain instability, and the brackets $\langle \ \rangle$ denote the expectation value of the time average. We further assume that the distribution of the noise N is identical in all images and that there are no noise correlations between different images: $\langle N_{i,k} \rangle = 0$, $\langle N_{i,k} N_{i,l} \rangle = 0$, and $\langle f(\alpha_k) g(N_{i,k}) \rangle = \langle f(\alpha_k) \rangle \langle g(N_{i,k}) \rangle$, where $N_{i,k}$ is the noise in pixel *i* of image *k*, and *f* and *g* are arbitrary functions.

In our model, the value of pixel i in image k is $S_{i,k} = (1+\alpha_k)(S_0 + N_{i,k}) = (S_0 + \alpha_k S_0 + N_{i,k} + \alpha_k N_{i,k})$. Note that the deviation from the "true" value contains contributions from both the thermal noise *N* and the gain instability α. We wish to compute the signal to noise ratio as a function of ROI size. A square ROI with side length *n* contains n^2 pixels. The expectation value of the time average signal $S_k(n)$ in such an ROI is $\langle S_k(n) \rangle = \left\langle \sum_{i=1}^{n^2} (S_0 + \alpha_k S_0 + N_{i,k} + \alpha_k N_{i,k}) \right\rangle = n^2 S_0$, and the expectation value of the mean square signal in the ROI is $\langle (S_k(n))^2 \rangle = \left\langle \left(\sum_{i=1}^{n^2} (1+\alpha_k)(S_0 + N_{i,k}) \right)^2 \right\rangle$, which, using the statistical properties of the random variables, the fact that $S_0 >> \sigma_N, \sigma_g$, and dropping terms of order σ^4 , can be rewritten as $\langle (S_k(n))^2 \rangle \cong n^4 {S_0}^2 + {\sigma_g}^2 n^4 {S_0}^2 + n^2 {\sigma_N}^2$.

The reciprocal of the signal-to-noise ratio is therefore

$$1/TSNRn = \frac{\sqrt{\langle (S_k(n))^2 \rangle - \langle (S_k(n)) \rangle^2}}{\langle (S_k(n)) \rangle} = \frac{\sqrt{n^4 {S_0}^2 {\sigma_g}^2 + n^2 {\sigma_N}^2}}{n^2 S_0} = \frac{\sqrt{1 + n^2 {\sigma_g}^2 ({S_0}^2 / {\sigma_N}^2)}}{n (S_0 / \sigma_N)} . \qquad [2]$$

Equation 2 is equivalent to Equation 1 if SNR= S_0 / σ_N and λ=σ_g.

Note that the validity of Equation 2 is not restricted to square ROIs; it is valid for any ROI of n^2 points selected from a uniform region of a phantom.

Results

To test our hypothesis that the deviation of 1/TSNR vs n from the expected 1/n dependence is caused by gain instability, we defined two semi-circular masks, called the reference mask and the ROI mask, on a times series of echo-planar images of a silicon oil phantom. We generated a Weiskoff plot by selecting pixels at random from the ROI mask. We then estimated the gain for each image by computing the average of signal in the reference mask and renormalized the data by dividing the ROI data for each image by the estimated gain of the image. The results are shown in Figure 1. The corrected data much more closely approaches the ideal 1/n behavior than does the uncorrected data. Residual deviation from 1/n behavior is probably caused by error in the gain estimates.

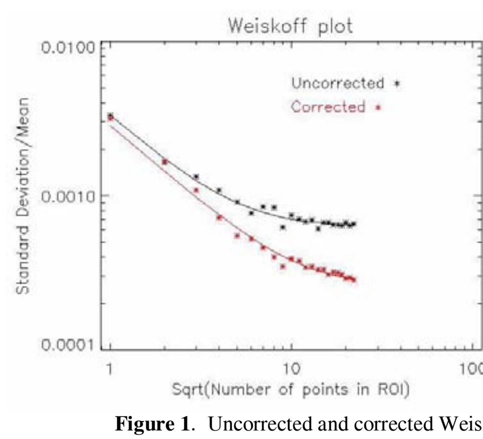

Figure 1. Uncorrected and corrected Weiskoff plots of data from on silicon oil phantom.

Conclusion.

We have shown that random fluctuations in gain result in Weisskoff EPI stability plots (1) having the functional form proposed by Bodurka (2). We provide physical justification for the previous heuristic model and show that the stability plots approach the expected 1/n behavior if the data are corrected for gain instability.

References:

1) Weisskoff R. MRM 36:643 (1996);
2) Bodurka et al. Proc. ISMRM 14:1094, (2006);