A level-set approach to joint nonlinear registration and segmentation using fast numerical scheme

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Image registration can assist image segmentation, and conversely, segmentation can assist registration [4]. A joint segmentation and registration of MR images of the same modality is considered. We provide an alternative approach to the one described in [5]. We also added a term to the objective function that improves the accuracy of registration of images of the same modality, and segmentation of the test image.

Method. In this study, to simultaneously segment and register an image pair the following objective function is minimized with respect to the unknown level set function ϕ [1] and displacement field $\mathbf{u}$

$$E(\phi,\mathbf{u}) = \int_\Omega f_{in} H(\phi) d\mathbf{x} + \int_\Omega f_{out}(1 - H(\phi)) d\mathbf{x} + \int_\Omega \hat{f}_{in}(\mathbf{x}+\mathbf{u}) H(\phi) d\mathbf{x} + \int_\Omega \hat{f}_{out}(\mathbf{x}+\mathbf{u})(1 - H(\phi)) d\mathbf{x} + \int_\Omega \left(I - \hat{I}(\mathbf{x}+\mathbf{u})\right)^2 d\mathbf{x} + R \qquad (1)$$

where I is the reference image, $\hat{I}$ - test image transformed according to displacement field $\mathbf{u}$. H denotes the Heaviside function. An argument of the functions f and ϕ is generally omitted, and present only when computed at a point different than $\mathbf{x}$. The functions are define as follows

$$R \equiv \mu \int_\Omega \delta(\phi)|\nabla\phi| d\mathbf{x} + \nu \int_\Omega \delta(\phi) d\mathbf{x} + \alpha \int_\Omega \Psi\left(\sum_{i=1}^{2} |\nabla u_i|^2\right) d\mathbf{x}, \quad f_{in} \equiv (I - m_{in})^2, \quad f_{out} \equiv (I - m_{out})^2, \quad \hat{f}_{in} \equiv (\hat{I} - \hat{m}_{in})^2, \quad \hat{f}_{out} \equiv (I - \hat{m}_{out})^2$$

where δ is the Dirac delta function, and m_{in}, m_{out} denote average intensities of the reference or test images in the regions where the level set function is nonnegative and negative, respectively. Ψ is a monotonic non-decreasing function; following [3] this is chosen to be $\Psi(s^2) = \varepsilon s^2 + (1-\varepsilon)\lambda^2 \sqrt{1 + \frac{s^2}{\lambda^2}}$. All integrals are computed in the coordinate system of the reference image. The first two terms are classical segmentation terms computed for the reference image [2]. The third and fourth are the segmentation terms computed for the test image. During the process of joint segmentation and registration the match between the transformed test and reference images is iteratively improved. Thus, the level set function ϕ used to segment the reference image is expected to reasonably well segment the registered test image. The next term, the mean square error (MSE) between the reference and transformed test images, is included to improve the quality of registration. The last term R contains regularization factors for segmentation and registration. Constants μ, ν and α are weights for the regularization terms. Smaller values of μ and ν allow finer features to be detected in the images during segmentation. A larger value of α forces a smoother displacement field during registration.

Minimizing the objective function E with respect to the level set function ϕ and displacement field $\mathbf{u}$ yield the following PDE system

$$\frac{\partial\phi}{\partial t} = -\delta(\phi)(f_{in} - f_{out}) - \delta(\phi)\left(\hat{f}_{in}(\mathbf{x}+\mathbf{u}) - \hat{f}_{out}(\mathbf{x}+\mathbf{u})\right) + \mu\delta(\phi)\nabla\cdot\frac{\nabla\phi}{|\nabla\phi|} - \nu\delta(\phi) \qquad \frac{\partial u_j}{\partial t} = -\frac{\partial\hat{f}_{in}(\mathbf{x}+\mathbf{u})}{\partial u_j} H(\phi) - \frac{\partial\hat{f}_{out}(\mathbf{x}+\mathbf{u})}{\partial u_j}(1 - H(\phi)) - 2\left(\hat{I}(\mathbf{x}+\mathbf{u}) - I\right)\frac{\partial I(\mathbf{x}+\mathbf{u})}{\partial x_j} + \alpha\sum_{i=1}^{m}\frac{\partial}{\partial x_i}\left(\Psi'\frac{\partial u_j}{\partial x_i}\right)$$

Since the PDEs are coupled, we first solve the equation with respect to ϕ, then use the obtained ϕ to solve the second equation with respect to $\mathbf{u}$, etc. The PDE system is solved using the AOS scheme [3]. We considered two objective functions: eq (1), denoted by E_1 below, and eq (1) without the MSE term, denoted by E_2.

Results. An MR image of a tomato was used in this study. After the first image was acquired, the tomato was locally distorted and another image acquired. The software is implemented for 2D images; thus a single slice (256x256 pixels) of the images was used. The maximal displacement between two images is approximately 20 pixels. First we performed segmentation and registration using objective function E_1. For some selected values of weights and time steps the obtained segmentation and registration is presented in the row 1 of the figure below. From left to right: reference image, unregistered test image, and registered test image (all overlaid with the contour corresponding to the zero level set), the difference between the reference and test images before and after joint segmentation and registration. One can observe that the reference image is accurately segmented. The match between the test and reference images is significantly improved after registration, thus the registered test image is also accurately segmented. Small features in the images are not registered, because a large weight assigned to the regularization term forced it to find a smooth displacement field. Similarly, the results obtained by minimizing function E_2 using same values of parameters are shown in row 2 of the Figure. Comparing these two results indicates that adding the MSE term produces a better match between the images, and increases the accuracy of segmentation of the test image.

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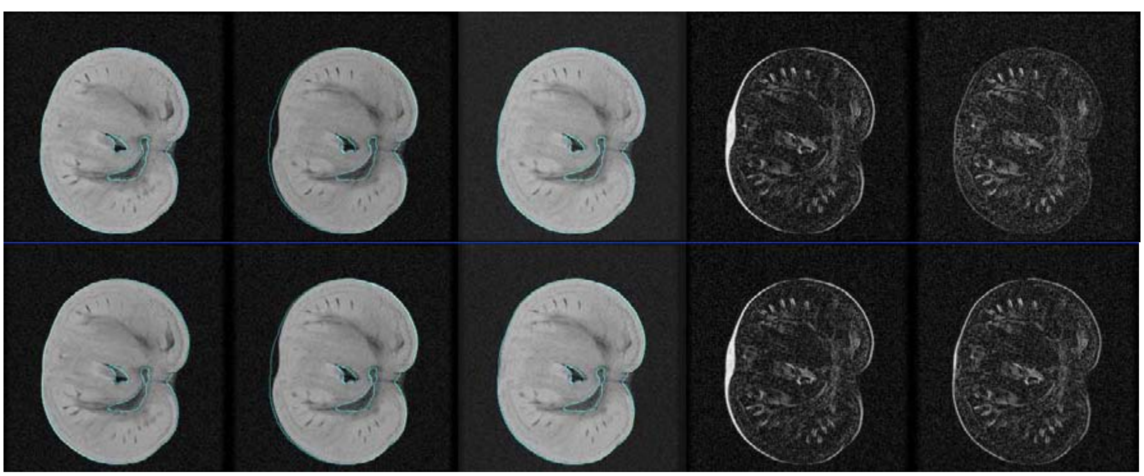

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