A Simple Method for Measuring and Removing Susceptibility Artifacts

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Introduction: Small variations in magnetic susceptibility and the physical/chemical environment can lead to variations in the static field strength and the spin-spin decay rate within a sample. These variations, in turn, lead to image artifacts, which it may be desirable to remove. In this abstract we describe a method, using two acquisitions at different gradient strengths, for removing these artifacts and mapping the variations of the B_0-field and R_2. For simplicity of exposition we describe the method in the context of radial sampling.

Theory: Let ρ(x) denote the proton density within a 2d-slice of a sample, $\Delta B(x)$ the variation in the static field strength and $\Delta R_2(x)$ the variation in the spin-spin relaxation rate around the mean $\bar{R}_2$ of $R_2(x)$, with respect to the density $\rho(x)dx$. Suppose that the slice is selectively excited and, at the conclusion of the rephrasing step, we apply a frequency encoding gradient, $g(\cos\theta, \sin\theta)$, to acquire samples of $\hat{\rho}(k)$ along a radial like in *k*-space. If acquisition begins at *t*=0, then the signal at time *t* is given by: $S(t) = e^{-\bar{R}_2 t} \iint e^{-i\langle x, k(t)\rangle} e^{-t(i\gamma\Delta B(x) + \Delta R_2(x))} \rho(x)dx$, where $k(t) = \gamma g t(\cos\theta, \sin\theta)$. We can express $t\gamma = g^{-1} |k(t)|$ and therefore:

$S(t) = e^{-\bar{R}_2 t} \iint e^{-i\langle x, k(t)\rangle} e^{-|k(t)| g^{-1}(i\Delta B(x) + \gamma^{-1}\Delta R_2(x))} \rho(x)dx$. If we assume that ΔB and ΔR_2 are small, or at least, that $|k(t)|$ is small, then we can approximate the second exponential using $e^x \approx 1 + x$ to obtain that

$S(t) \approx e^{-\bar{R}_2 t} \iint e^{-i\langle x, k(t)\rangle} [1 - |k(t)| g^{-1}(i\Delta B(x) + \gamma^{-1}\Delta R_2(x))] \rho(x)dx$. Thus we see that

$S(t) \approx e^{-\bar{R}_2 t} [\hat{\rho}(k(t)) - g^{-1} |k(t)| \hat{e}(k(t))]$ where $e(x) = (i\Delta B(x) + \gamma^{-1}\Delta R_2(x))\rho(x)$ is the error introduced by variability of B_0 and T_2. In the last formula we see that the error term depends linearly on g^{-1}. Hence, if we acquire samples of $\hat{\rho}(k)$, at the same points in k-space, with two different gradient strengths, then we can combine weighted versions of the two measurements to obtain samples of $\hat{\rho}(k)$ with the error term removed. The calculations here assume a radial sampling scheme; similar considerations apply to other sampling schemes. This technique can be used as the basis for a recursive algorithm to map the variations in R_2 and B_0.

Methods: (a) Two gradient strengths and radial sampling: Suppose that we measure each ray in *k*-space at two gradient strengths, $g_1 < g_2$, the samples are at taken at angles $\{\theta_k\}$, and frequencies $\{k_j\}$. The sampling times are $\{t_j^1 = j\Delta t_1\}$ and $\{t_j^2 = j\Delta t_2\}$; the measurements are given by

$m_{jk}^1 = e^{-t_j^1 \bar{R}_2} [\hat{\rho}(k_j\omega(\theta_k)) - g_1^{-1} |k_j| \hat{e}(k_j\omega(\theta_k))]$ and $m_{jk}^2 = e^{-t_j^2 \bar{R}_2} [\hat{\rho}(k_j\omega(\theta_k)) - g_2^{-1} |k_j| \hat{e}(k_j\omega(\theta_k))]$, where $\omega(\theta) = (\cos\theta, \sin\theta)$. The combination:

$M_{jk} = (g_2 - g_1)^{-1} [e^{-\bar{R}_2 t_j^1 g_2^{-1}(g_2 - g_1)} g_2 m_{jk}^2 - g_1 m_{jk}^1] = e^{-t_j^1 \bar{R}_2} \hat{\rho}(k_j\omega(\theta_k))$ gives the measurements at gradient strength g_1 uncorrupted by variations in either the static field strength or the spin-spin relaxation time. Applying the inverse Fourier transform gives ρ. The Fourier transform of the error term is obtained from the differences divided by the frequency: $\hat{e}_{jk} = |k_j|^{-1} (M_{jk} - m_{jk}^1) = e^{-t_j^1 \bar{R}_2} g_1^{-1} \hat{e}(k_j\omega(\theta_k)), j \neq 0$; at zero frequency we set $\hat{e}_{0k} = 0$. Applying the inverse Fourier transform to this data gives samples of $\rho(x)[\gamma^{-1}\Delta R_2(x) + i\Delta B(x)]$. Dividing by $\rho(x)$ gives a map of $\gamma^{-1}\Delta R_2(x) + i\Delta B(x)$. This procedure may used iteratively to get increasingly accurate estimates of the spin density and these variations. If $\Delta R_2(x)$ and $\Delta B(x)$ are not that small, then this procedure could be applied using a cutoff at higher frequencies, to remove, and map the low frequency variations in these quantities.

(b) SNR effects: The SNR of the combined (independent) measurements is approximately half the SNR of the measurements at the higher gradient strength. By using different amounts of signal averaging the two sets of measurements can be normalized to have the same SNR. In this case the overall loss of SNR is a factor of $\sqrt{5}$.

Experimental results: The data is acquired with a 3D hybrid radial pulse sequence. In-plane resolution is 150x150 microns with a slice thickness of 1.5 mm. Acquisition time for 8 slices is 5 minutes. Data is acquired at two gradient strengths 3.5 and 7 mT/m. A custom built surface coil is used to scan an *ex-vivo* tibia bone specimen, with a T_2 of about 40ms, in a 1.5T Siemens Sonata scanner. At the lower bandwidth, the acquisition of a ray in *k*-space requires about 20ms. The raw data are combined (without correcting for spin-spin decay) as 2*B-A, where A and B are the data sets acquired at the two gradient strengths. The improvement in the sharpness of the image, across the field of view, is quite apparent in Fig. 3.

Conclusion: We demonstrate a simple method for removing artifacts in MR-images due to small spatial variations in magnetic susceptibility and spin-spin decay rates.

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Fig. 1: Low bandwidth

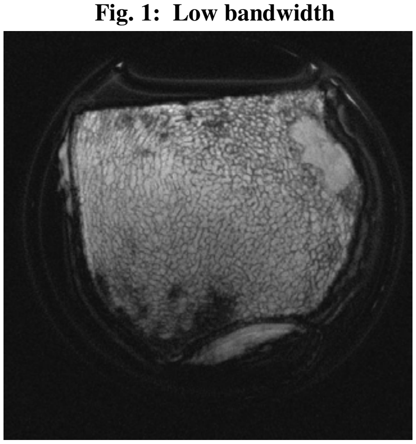

Fig. 2: High bandwidth

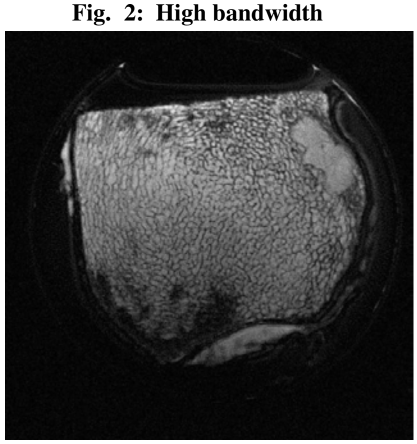

Fig. 3: Combined