Surpassing Square-Root of Imaging Time Accuracy Gain in T_1 Estimation using SPGR sequence.

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INTRODUCTION. The intrinsically low signal-to-noise-ratio (SNR) of MR images leads to a variability in the quantitative metric derived from them, limiting its usefulness. Many studies have been devoted to optimizing T_1 measurement strategies (1,2,3). All, however, share several limitations: 1) the choice of acquisition parameters is restricted in one way or another; 2) the effects of the MR signal decay during the readout are ignored; 3) and most important, the restriction of total imaging time is disregarded.

THEORY. In the spoiled gradient-recalled echo (SPGR) sequence, the image intensity, ρ, is related to effective transverse relaxation time T_2^*, spin density, ρ_0, flip angle, α, echo time, T_E, and repetition time, T_R, by Eq.1. Given two images, ρ_1, ρ_2, obtained with different acquisition parameters, T_1 can be estimated from ρ_1/ρ_2 numerically.

$$\rho = \rho_0 \frac{1-e^{-T_R/T_1}}{1-e^{-T_R/T_1}\cos(\alpha)}\sin(\alpha)e^{-T_E/T_2^*} \qquad [1]$$

$$\sigma_{T_1}^2 = \sigma_0^2(T_{adc})\left[\frac{1}{N_1}\left(\frac{\partial T_1}{\partial \rho_1}\right)^2 + \frac{1}{N_2}\left(\frac{\partial T_1}{\partial \rho_2}\right)^2\right] \qquad [2]$$

If $\sigma_0(T_{adc})$ is the standard deviation of the random noise in an individual image, determined by a common sample, instrument, readout duration T_{adc} ($T_{adc} \le 2T_E$) $\sigma_0(T_{adc}) \sim 1/\sqrt{T_{adc}}$ (5), field of view (FOV) and resolution then, using error propagation arguments (6), the variance σ_{T1} in T_1 (Eq.2) can be minimized subject to the total imaging time constraint $T=N_1T_{R,1}+N_2T_{R,2}$ where N_1 and N_2 are the number of averages of each experiment in the pair. For a given total experimental time T this minimization can be performed to yield the best acquisition protocol. (Optimality of protocols requires T_{adc} be as close to T_2^* as possible.) Comparing the accuracy gain to the square-root of imaging time, the optimal protocol leads to errors which decline much faster (see Fig.1).

Examples. For total imaging time of T=0.01T_1, the optimal protocol is identical to that of DESPOT1 (2): $T_{R,1}=T_{R,2}=0.005T_1$, α_1=3°, α_2=13°, $N_1=N_2=1$. If more time is available T=0.05T_1, according to (2) only the number of averages is changed $N_1=N_2=5$ which leads to $\sqrt{5}$ improvement in accuracy. For equal imaging time $T_{R,1}=0.025T_1$, $T_{R,2}=0.025T_1$, α_1=5°, α_2=30°, $N_1=N_2=1$ leads to $1.7\sqrt{5}$–$1.9\sqrt{5}$ accuracy improvement. In other words, accuracy accrues faster than in proportion to the square-root of imaging time (Fig.1a). Even though the protocol was optimized for an assumed value of T_1, it outperforms the referenced one between 0.01T_1 and 10T_1. Similar situation occurs with the protocol of Wang, *et al* (4) where $T_{R,1}=T_{R,2}$, $N_1=N_2$ restrictions were artificially imposed. Due to these restrictions, the T_1-error is 25% larger than achievable in the same imaging time (see Fig.1b). In this case the T_1-interval over which our protocol outperforms the reference is between 0.01T_1 and 2.16T_1 around the assumed T_1 value.

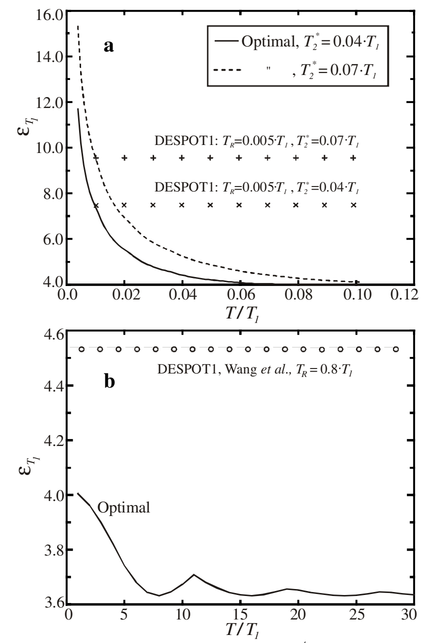

Fig.1 *The normalized error $\varepsilon_{T1}=\sigma_{T1}\sqrt{T}$ as a function of imaging time T. Top: For $T < 0.1\cdot T_1$ and representative $T_2^*=0.04\cdot T_1$ and $0.07\cdot T_1$ (solid and dashed lines). Bottom: For $T > T_1$, a regime where ε_{T1} no longer depends on T_2^*. Note that since the number of averages N_1 and N_2 are integer, ε_{T1} attains oscillatory behavior as a function of T. Also note that repeating the imaging pair until all time is spent, as prescribed by Deoni et al. and Wang et al. , leads to constant ε_{T1}, marked by "+"s, "×"s and "○"s.*

METHODS. The optimal protocol was verified experimentally on a 3T Siemens Trio whole-body imager (Siemens AG, Erlangen, Germany) using its transmit-receive body coil, on a uniform 15cm diameter, 40cm length cylindrical water phantom of $T_1 \approx$ 300ms using a 3D SPGR with 192×192×48 mm^3 FOV and 64×64×16 matrix. The acquisition parameters were according to DESPOT1 and as described here. Both experiments were done five times back-to-back using equal measurement time. Standard deviations of T_1 were computed on a pixel-by-pixel basis and their distributions were produced.

RESULTS. As predicted, when the new method is used, the standard deviations cluster closer to zero (see Fig.2). Based on the Kolmogorov-Smirnov test, the probability that the observed difference in distributions is accidental is less than 10^{-5} (7).

CONCLUSION. It is shown that the total imaging time uniquely determines the best achievable precision and a protocol to obtain it. While the SNR of the average individual images *does not* improve beyond the square-root of their acquisition duration, the precision of T_1 values derived from their combinations – does.

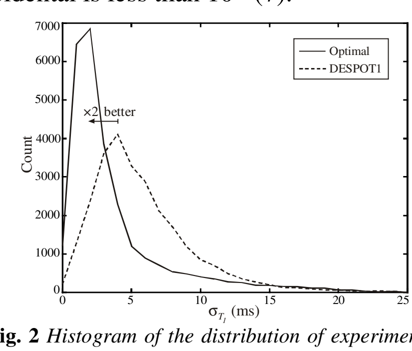

Fig. 2 *Histogram of the distribution of experimental variability σ_{T1} in T_1 estimates using DESPOT1 (dashed curve) and the proposed protocol (solid curve). Both experiments took equal time. Note that σ_{T1}'s produced by the proposed method cluster closer to zero than DESTPOT1's, indicating that smaller errors, (higher precision) occur more often.*

References.
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