An Optimal Framework for T1 Estimation in An SPGR Acquisition

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INTRODUCTION The longitudinal relaxation time, T_1, and the magnetization at thermal equilibrium, M_0, can be estimated from two or more spoiled gradient recalled echo (SPGR) images acquired with different flip angles and/or repetition times (T_R) [1-5]. To date, several approaches have been proposed for selecting the combination of number of data points, flip angles, and T_R values that would provide the best estimate (i.e. lowest variance) of a given T_1 [1-5]. These previous studies converge to the conclusion that an optimal approach is a two-point acquisition with constant T_R, and two flip angles yielding signal equal to $1/\sqrt{2}$, (≈70.7%) of the signal at the Ernst angle [2]. Numerical verification of this fact was shown by Deoni *et al*. [4]. However, methods that provide an optimal estimation of a single T1 are not ideally suited for studying the brain and other biological tissue that present a range of T1 values. We argue that to find optimal acquisition parameters for a range of T_1 values, it is necessary to take M_0 into account since each voxel in the brain contains different pair of M_0 and T_1. No previous studies have attempted to optimize acquisition parameters for M_0. Here, we propose a framework for finding a set of optimal flip angles by minimizing the variance of T_1 weighted by the joint density of (M_0 , T_1) at a single T_R.

METHODS The nonlinear least squares objective function for T_1 estimation can be written as: $f = \frac{1}{2}\sum\left(s_i - M_0 \sin(\alpha_i)\frac{1-\exp(-T_R/T_1)}{1-\cos(\alpha_i)\exp(-T_R/T_1)}\right)^2$ where s_i are the observed signals, α_i are the flip angles, M_0 is the unknown equilibrium longitudinal magnetization, T_R is the repetition time, and T_1 is the unknown longitudinal relaxation time. The variance of T_1 can be shown to be: $\sigma^2_{T_1}(M_0, T_1, T_R, \{\alpha_i\}) = \frac{\sigma^2 T_1^4}{\xi^2(\xi-1)T_R^2 M_0^2}\sum_i(\sin(\alpha_i)/(\xi-\cos(\alpha_i))^2 / \sum_i\sum_j A_{ij}$ where $A_{ij} = \frac{\sin^2(\alpha_i)\sin^2(\alpha_j)(\cos(\alpha_j)-1)(\cos(\alpha_j)-\cos(\alpha_i))}{(\xi-\cos(\alpha_i))^3(\xi-\cos(\alpha_j))^4}$, $\xi = \exp(T_R/T_1)$ and σ is the noise SD. The proposed strategy for selecting a set of optimal flip angles, $\{\alpha_i\}$, is by minimizing the sum of all variances of T_1 within the brain weighted by the joint density of (M_0 , T_1), $f(m_0, \tau_1)$; this objective function can be expressed as: $\sum_{(m_0,\tau_1)\in\Omega}\sigma^2_{T_1}(m_0, \tau_1, T_R, \{\alpha_i\})f(m_0, \tau_1)$ where Ω is the region of interest, e.g. the whole brain, the whole white matter, or any particular region.

RESULTS AND DISCUSSION We tested our approach using SPGR acquisitions in the human brain of healthy volunteers. First we computed M_0 and T_1 maps from two-point SPGR images that were optimized for an assumed T1 of 1200 ms according to Wang [2] ($\alpha_1 = 3^\circ$, $\alpha_2 = 17^\circ$, T_R=8.6 ms). Then we computed the marginal histograms of T_1, M_0, and the smoothed joint histogram of T_1 and M_0, which are shown respectively in Fig. 3A-3C, for the brain shown in Fig. 1 and 2.

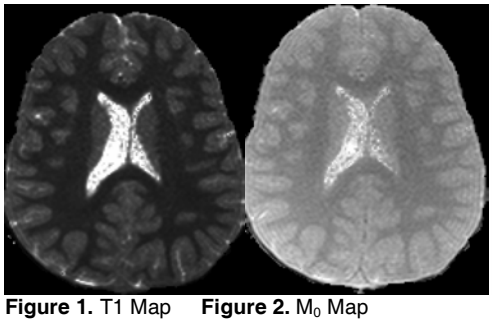

Figure 1. T1 Map **Figure 2.** M_0 Map

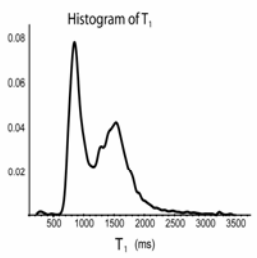

Figure 3A. Histogram of T1

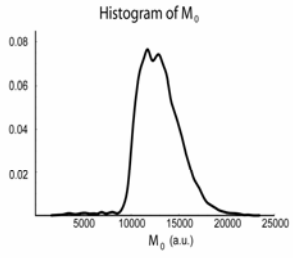

Figure 3B Histogram of M_0

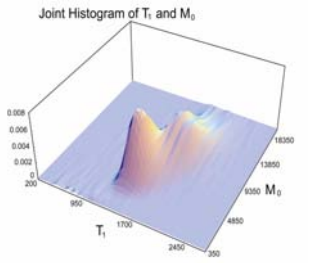

Figure 3C. Joint Histogram of T1 and M_0

Finally, we recomputed optimal angles with the strategy proposed above. In the example presented, at TR=10 ms, we obtained the following optimal angles : (2.83°, 16.48°), (3.08°, 18.30°, 18.30°), (2.83°, 2.83°, 16.48°, 16.48°), (2.98°, 2.98°, 17.48°, 17.48°, 17.48°) and (2.83°, 2.83°, 2.83°, 16.48°, 16.48°, 16.48°) for a 2-,3-,4-, 5- and 6-point acquisitions, respectively. The mean value of T1 over the entire brain excluding the lateral vectricles was about 1279 ms. The first finding is that for multiple-point acquisitions, the optimal solution is represented by pairs of angles, rather than by a range of angles, as one may have expected. In particular, acquisitions with even number of points, are essentially constructed by "evenly" replicating the two fundamental angles from the two-point acquisition, a finding in line with that of Wang et al.[2] found for optimizing a single value of T1. The second finding is that the pairs of angles found here would have been optimal for a single T1 at about 1389 to 1405 ms, a value much higher than the average value of T1 in the brain studied.

CONCLUSION

We have presented a simple framework for finding optimal flip angles in computing T_1 from SPGR images that is weighted by the joint density of (M_0 , T_1) at a single T_R. Our results suggest that when the proposed optimal acquisition strategy is applied to imaging tissues with a range of T_1 and M_0 values, it is optimal, in the sense of having lower overall variance of T_1, to replicates "evenly" the two fundamental angles in a two-point acquisition — as in the case of a single T1. However, the angles should be set for a T1 higher than the average T1 of the tissue. We believe that our approach represents a first step in defining optimal acquisition parameters for clinical MRI studies aimed at assessing a range of T1 values in tissues from SPGR signals.

REFERENCES

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