Dealing with spatially varying noise in T_2* mapping with SENSE

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Introduction: Relaxation time constants such as T_2* allow to quantitatively analyze tissues and to measure in-vivo contrast agent concentrations. They can be computed on the basis of a multi-echo measurement by fitting an exponential function of the type $s_0\exp(-t/T_2^*)$ to the magnitude data. However, least-squares algorithms, which have been proposed to perform the fit [1, 2], tend to overestimate the true relaxation time, because they do not take into account the non-Gaussian noise statistics of magnitude data [3]. As a consequence, an SNR-dependent bias is spuriously introduced, which may hamper the comparison of relaxation time constants between voxels having different noise levels. In this work, the case of T_2* mapping in combination with SENSE, which is characterized by spatially varying SNR, is investigated. A maximum likelihood algorithm is applied to obtain non-biased estimates of the relaxation time.

Theory: The proposed fitting algorithm applies a maximum likelihood approach to compute the optimal relaxation parameters (s_0, T_2*) given a time series of N signal samples s_k measured at echo times TE_k. Since magnitude data follow a non-central chi distribution, the likelihood function of (s_0, T_2*) can be expressed as [3]:

$$L(s_0, T_2^*) = \prod_{k=1,N} \frac{s_k}{\sigma^2} \exp\left(-\frac{s_k^2 + \bar{s}_k^2}{2\sigma^2}\right) I_0\left(\frac{s_k \bar{s}_k}{\sigma^2}\right),\ \bar{s}_k = s_0 \exp\left(-\frac{TE_k}{T_2^*}\right). \qquad (1)$$

I_0 is the modified Bessel function of the first kind, of degree 0, and σ is the voxel-dependent noise standard deviation. The parameters (s_0, T_2*) are found by maximizing the likelihood function L. Powell's method [4] can be applied to solve this non-linear optimization problem.
In SENSE, the noise level in the reconstructed images depends on the sensitivity profile C and increases with the acceleration factor R. Minimal noise propagation is achieved when the SENSE reconstruction is based on the noise covariance matrix Ψ of the input data. After reconstruction, the standard deviation for an unfolded voxel is then given by [5]:

$$\sigma = \sqrt{R\left(C^H \Psi^{-1} C\right)^{-1}}\ . \qquad (2)$$

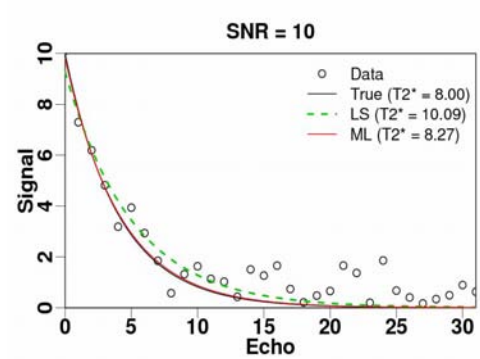

Fig.1: Exponential fits obtained with the simulated data.

Methods: Simulations with different values of T_2* and of the SNR were performed to evaluate the proposed fitting algorithm. Then, a phantom consisting of 4 probes filled with different concentrations of a super-paramagnetic iron oxide (SPIO, Resovist® Schering AG) was imaged with a custom-made 6 element head-coil array on a 1.5T scanner (Achieva, Philips Medical Systems). A reference T_2* scan with a high SNR and no acceleration (R=1) was made; then, a four-fold accelerated scan was performed (R=4). Both experiments consisted of 32 echoes with an echo spacing equal to 2 ms. The following parameters were used: resolution 1.0×1.0×5.0mm, TR=120ms, flip angle=30°. All images were reconstructed with the SENSE algorithm. The noise covariance matrix Ψ was assessed on the basis of calibration noise data. The standard deviation maps of the reconstructed images were computed according to Eq. 2. T_2* maps were calculated for each dataset, with both a least-squares (LS) algorithm and the proposed maximum likelihood (ML) algorithm.

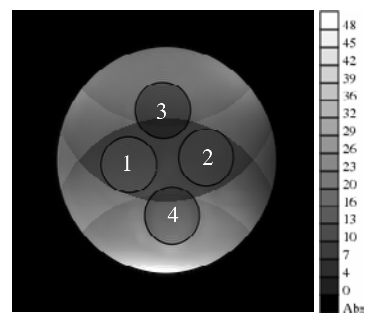

Fig. 2: Map of the SNR corresponding to the four-fold accelerated SENSE reconstruction.

Results: Fig. 1 shows an example of exponential fits obtained with the simulated data for values of the SNR and the relaxation time similar to those encountered in the phantom experiment. When the SNR is low, the fit obtained with the least squares algorithm is significantly perturbed by the noisy samples located at the tail of the curve. This effect is avoided with the maximum likelihood algorithm.
Fig. 2 shows the SNR map corresponding to the four-fold SENSE reconstruction: it has a highly spatially varying pattern. The combination of low SNR and short T_2* values for the probes 1 and 2 results in a significant bias in the least-squares estimation of T_2* (Tab. 1). The relative error equals 23.3% for probe 1 and 21.9% for probe 2. This bias is largely reduced when applying the maximum likelihood algorithm, with relative errors equal to 1.8% and 2.7%, respectively. For higher values of T_2*, as found in probes 3 and 4, the two algorithms yield similar estimates, as expected, with the maximum likelihood algorithm achieving a slightly smaller relative error.

	Probe 1	Probe 2	Probe 3	Probe 4
Ref.	7.44	11.78	141.59	44.31
SENSE R=4 – LS	9.17	14.36	155.68	45.50
SENSE R=4 – ML	7.58	12.10	153.00	45.21

Tab. 1: Mean T_2* values for each probe.

Discussion and conclusion: The proposed algorithm significantly reduces the systematic fitting error due to non-Gaussian noise statistics, especially for low SNR and for species with short T_2*. The estimation of the relaxation time becomes more robust to noise and less sensitive to the number of echoes used. Averages computed over a region of interest are not biased, which enables the comparison of relaxation time values even in the presence of spatially or temporally varying SNR. This issue is especially relevant for fast scanning techniques like SENSE that suffer from strong local noise amplification for high acceleration factors. Applications requiring short scan times, e.g. to avoid long breathholds like T_2* relaxometry of the liver, may benefit from this computation method. The method applies also to T_2 and T_1 mapping.

References:
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