Iterative Fourier Transform Magnetic Resonance Current Density Imaging (FT-MR-CDI)

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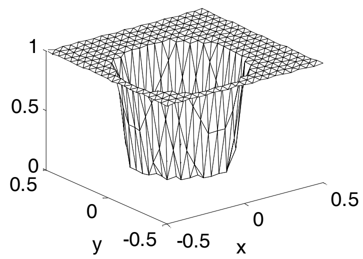
Introduction:

Magnetic resonance current density imaging (MR-CDI) and magnetic resonance electrical impedance tomography (MR-EIT) are dual problems in that the solution of one makes the solution of the other trivial. In this study the MR-CDI problem is tackled using an iterative Fourier Transform (FT) method. The magnetic field generated by the internal currents of a conducting object has three components but only its z-component, H_z, is measured in the object by an MR scanner if subject rotations are to be avoided. MR-CDI and MR-EIT are inherently 3D problems when applied to human imaging. However 2D applications may also be of interest for sample analysis where preparation of a slice object is possible. In this study 2D simulation results for the proposed algorithm for MR-CDI are presented.

The algorithm:

Let Ω be a connected and bounded domain in the xy-plane, representing a biological tissue with positive non-zero nonuniform conductivity σ. We assume that current is either injected into Ω by surface electrodes or induced in it by an external AC magnetic field. The magnetic field due to the internal current density $\mathbf{J} = (J_x\ J_y)$ has only H_z component in Ω, which is related to J_x and J_y by the Biot–Savart integral. It has been shown that the $\boldsymbol{F}\{J_x\} = (2jk_y/k)\boldsymbol{F}\{H_z\}$ and $\boldsymbol{F}\{J_y\} = (-2jk_x/k)\ \boldsymbol{F}\{H_z\}$ where $\boldsymbol{F}$ is the Fourier Transform operator, k_x, k_y are are spatial frequencies in cycles/length in x and y directions respectively, and $k = \sqrt{(k_x^2 + k_y^2)}$. These filters are null at the origin and their derivation assumes divergence free current in Ω.

Figure 1. Conductivity



Although J_x and J_y are confined to Ω, H_z is not and therefore in order to find its FT it must be measured in a sufficiently large region Ω_2 covering Ω. However practically, using MRI, one can measure H_z only in the tissue domain Ω. In this study we have developed an iterative method whereby the problem is solved by using H_z measurements in Ω only. The algorithm is as follows: 1) H_z is assumed to be zero in $\Omega_2\backslash\Omega$, and is assigned the measured values in Ω . 2) FT of H_z is is calculated in Ω_2, and using the above inverse filters, J_x and J_y are calculated in Ω_2. 3) The calculated currents are then multiplied by a support function which is 1 in Ω but 0 in $\Omega_2\backslash\Omega$. From this corrected (modified) current density, H_z is calculated back in Ω_2. 4) The values calculated for H_z in $\Omega_2\backslash\Omega$ are retained, but the values calculated in Ω are replaced by the measured H_z values and step 2 jumped at. The algorithm has a good convergence behavior, and it finds the unmeasured values of H_z in $\Omega_2\backslash\Omega$ as well as calculating the current density in Ω. In fact by way of the determination of H_z in $\Omega_2\backslash\Omega$ in addition to its measurement in Ω , the reconstructed current density is forced to lie in Ω only.

Simulation results:

Figure 1 shows the conductive object defined in $\Omega = \{-0.5 \le x \le 0.5,\ -0.5 \le y \le 0.5\}$, and the assumed σ distribution. The Finite Element Method is used to solve for the current density $\mathbf{J}$ in Ω for when current is injected from the x = -0.5 edge and removed from the x = 0.5 edge. Current is also calculated for the case of uniform conductivity distribution in the object. This current, called $\mathbf{J}_{uni}$, is independent of the uniform conductivity value used. The difference current, $\mathbf{J}_d = \mathbf{J} - \mathbf{J}_{uni}$ is divergence free and is used as our source current for simulation purposes. The magnetic field generated by this difference current is then calculated in Ω to be used as the measured magnetic field. Reconstructions are made for $\Omega_2 = \{-0.7 \le x \le 0.7,\ -0.7 \le y \le 0.7\}$. The reconstructed current density is shown in Figure 2, and the reconstructed magnetic field is shown in Figure 3. It is observed that H_z has nonzero values in $\Omega_2\backslash\Omega$, and however reconstructed current is almost null in $\Omega_2\backslash\Omega$. Results shown in Figures 2 and 3 are for the 6th iteration at which the current density is within the correct distribution with less than 5% relative norm error.

Figure 2. Reconstructed Current Density

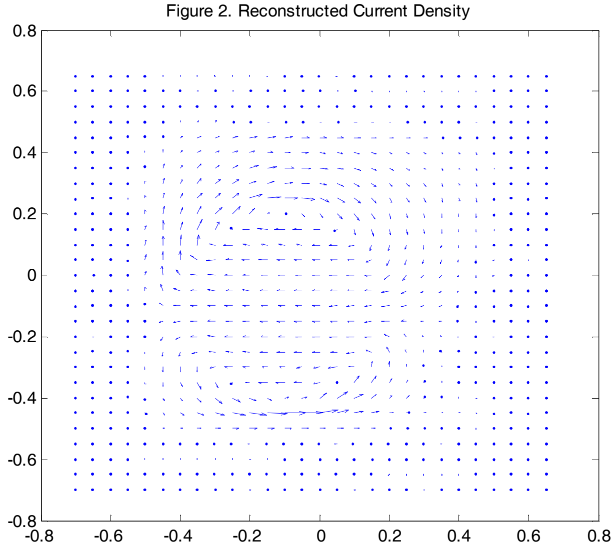
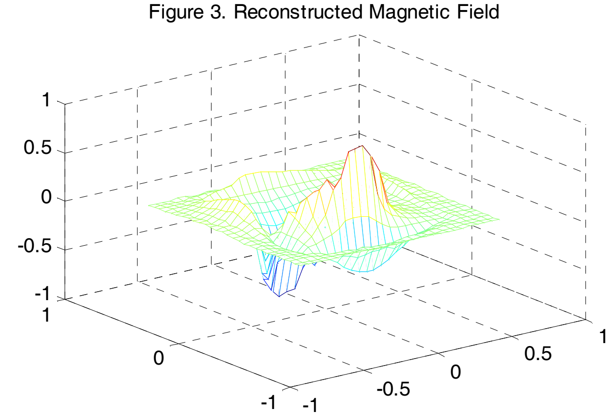


Figure 3. Reconstructed Magnetic Field



Conclusions:

The proposed FT based algorithm for 2D MR-CDI appears to have good convergence behavior and it is also fast due to the fast implementation of the FT. Performance of the algorithm against noise is expected to be good in view of the fact that rejection of high frequency noise in H_z can be easily done in the Fourier domain. Note that the proposed algorithm is also applicable to induced current MR-EIT systems because induced currents are by nature divergence free.