

# Intrinsic Signal Amplification in 2D SENSE Elliptical Centric 3D Contrast-Enhanced MRA

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## Introduction:

It is well accepted that the signal-to-noise ratio (SNR) of SENSE [1] is reduced by the factor  $1/(gR^{1/2})$  in comparison to a reference scan with the same spatial resolution. This analysis assumes that the magnetization being imaged is constant over the scan. Other works have studied the application of 1D parallel imaging to non-constant magnetization and noted that net spatial resolution can potentially improve if acceleration is used [2,3]. The purpose of this work was to extend this analysis to 2D SENSE [4] applied to 3D contrast-enhanced MR angiography (CE-MRA) using the elliptical centric (EC) view order. We show that there is an intrinsic signal amplification effect due to 2D SENSE encoding of the waning magnetization. This compensates in part for the traditional  $1/(gR^{1/2})$  SNR loss. The effect is shown analytically and then demonstrated in a study of whole brain MR venography done in 20 volunteers using R=4 2D SENSE.

## Methods:

**Analysis.** The derivation of the signal amplification effect in 2D SENSE follows directly from the analysis of the spatial resolution limit of EC CE-MRA [5]. This is presented in the equations shown in Fig. 1. One starts with a presumed time-varying contrast bolus, an example of which is the gamma variate of Eq. 1 where  $\tau$  is a time constant. The CE-MRA scan is initiated some time after bolus arrival. This can be expressed by creating a time-shifted version of the original bolus (Eq. 2), where  $f$  is a dimensionless parameter. The EC view order then maps the time-varying contrast to  $k_y$ - $k_z$  space at increasing distances from the k-space origin. This association of elapsed time  $t$  with k-space radius  $k$  is expressed in Eq. 3. Substitution of  $t$  (Eq. 3) into Eq. 2 converts the bolus signal curve into the k-space modulation function  $H(k)$  (Eq. 4) where the expression of Eq. 5 is also used which describes the rate at which k-space is sampled. Here SENSE acceleration factors  $R_Y$  and  $R_Z$  along the  $k_y$  and  $k_z$  directions are allowed. Finally, Fourier transformation of Eq. 4 yields the point spread function  $h(r)$ , and when evaluated at the origin yields the signal amplitude of the reconstructed image,  $h(0)$ , Eq. 6. Note that the amplitude is proportional to the net SENSE acceleration  $R = R_Y \times R_Z$ . A simulated  $b(t)$  and two corresponding modulation functions  $H(k)$  are illustrated in Figs. 2A-B, for non-SENSE ( $R=1$ ) and SENSE ( $R=4$ ) acquisitions. A  $\tau$  of 20 sec and scan times of 120 sec and 30 sec were assumed. The ratio of areas under the two curves in Fig. 2B gives the signal amplification due to SENSE, numerically computed here as 2.18. This factor is less than  $R=4$  because of the limited acquisition time. Accounting for this effect yields the expression for the relative SNR in a SENSE acquisition (Eq. 7), where  $1 < A < R$ .

**Experiments.** The existence of a signal amplification  $A$  greater than unity was tested using whole brain EC contrast-enhanced MR venograms [6] at 1.5 T. Measurement of a bolus curve in the superior sagittal sinus (SSS) allowed generation of curves similar to those in Fig. 2B, resulting in a predicted signal amplification of 1.31. Next 20 volunteers were imaged, once with a reference non-SENSE scan and then with R=4 2D SENSE. Ten minutes were allowed between scans for contrast clearing. The order of scans was randomized from volunteer to volunteer. SNR values and g-factors were measured using small, 60 mm<sup>2</sup> regions within and near the anterior, superior, and posterior aspects of the SSS. From these the estimate for  $A$  was determined.

## Results:

Fig. 3 shows midline sagittal partitions of the non-SENSE (A) and SENSE (B) venograms from one volunteer. Table 1 shows results. For all three regions the measured signal amplification  $A$  is significantly larger than unity ( $p < 0.001$ ) and within one s.d. of the estimated value of 1.31.

## Conclusions:

The phenomenon has been described and demonstrated of a signal amplification effect which occurs when 2D SENSE is applied to elliptical centric contrast-enhanced MRA. This may allow higher acceleration factors than may seem plausible because of expected SNR loss.

## References:

- [1] Pruessmann K, MRM 42:952(1999); [2] Griswold M, MRM 47:1202 (1999); [3] Jaermann T, MRM 55:335 (2006); [4] Weiger M, MAGMA 14:10 (2002); [5] Fain S, MRM 42:1106 (1999); [6] Farb R, Radiology 226:203 (1999)

$$\begin{aligned} \hat{b}(t) &= \frac{1}{\tau^2} t^2 e^{-t/\tau} & (1) \\ b(t) &= \hat{b}(t + f\tau) = e^{-f} e^{-t/\tau} \left\{ \frac{t^2}{2\tau^3} + \frac{ft}{\tau^2} + \frac{f^2}{2\tau} \right\} & (2) \\ t &= \frac{\pi \cdot TR}{\Delta k_y \cdot \Delta k_z} k^2(t) & (3) \\ H(k) &= e^{-f} e^{-\frac{\pi k^2}{M\tau}} \left\{ \frac{\pi^2 k^4}{2M^2\tau^3} + \frac{k^2 f \pi}{M\tau^2} + \frac{f^2}{2\tau} \right\} & (4) \\ M &= \frac{(\Delta k_y \cdot R_y) \cdot (\Delta k_z \cdot R_z)}{TR} & (5) \\ PSF(0) &= h(0) = e^{-f} \cdot M = \frac{e^{-f} \cdot R_y \cdot R_z}{FOV_y \cdot FOV_z \cdot TR} & (6) \\ rSNR &= \frac{A}{(gR^{1/2})} & (7) \end{aligned}$$

Fig. 1. Derivation of signal amplification.

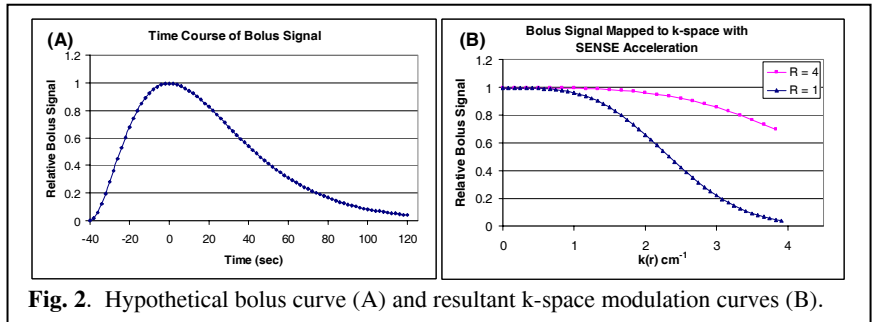


Fig. 2. Hypothetical bolus curve (A) and resultant k-space modulation curves (B).

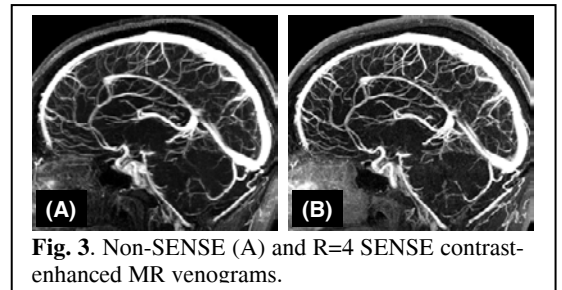


Fig. 3. Non-SENSE (A) and R=4 SENSE contrast-enhanced MR venograms.

**Table 1.** Summary of observed signal amplification factors ( $A$ ) for three regions of the SSS in  $n=20$  volunteers. All three results are significantly larger than unity ( $p < 0.001$ ). Expected  $A = 1.31$ .

	Anterior	Superior	Posterior
Mean g-factor	1.15	1.00	1.17
Observed $A$	$1.37 \pm 0.07$	$1.31 \pm 0.06$	$1.26 \pm 0.05$