Tikhonov regularization optimisation for PreLearn: effects on the detection of activations in functional MRI

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Introduction

PreLearn is a new method for image reconstruction in Parallel MRI already introduced in [1]. It performs the reconstruction in the image domain and the unfolding matrix is calculated by a linear 'learning-by-example' strategy. In [2] was shown that PreLearn respects the activations when applied to the reconstruction of functional MRI (fMRI) images. Linear learning is intrinsically an ill-posed problem. At reconstruction stage noise is amplified and the reconstructed images can present a poor signal to noise ratio. This decreases the number of activations detected by post-processing algorithms. In this abstract, we apply Tikhonov regularization to the PreLearn reconstruction of fMRI series. We optimise the Tikhonov regularization parameter for the detection of such activations.

Description of the method

PreLearn is based in the physical assumption that the reconstruction process is the same for a voxel and its neighbourhood. This assumption leads to a system of n_n equations, n_n being the number of neighbours plus one (the pixel itself), see [1] for more details. The unfolding matrix, N, can be estimated using a pseudo-inverse as:

$$N = R\,F^{H}\,(F\,F^{H})^{-1}, \quad (1)$$

where R is an n_r x n_n matrix of Reference intensities and F is an n_f x n_n matrix containing corresponding Folded pixels, n_r is the reduction factor and n_f is the number of coils used in the acquisition system. We note that R can be built from a reference image at full resolution and F from the folded images of the coils. When PreLearn is to be applied, we cannot forget that data used to build matrices R and F will be corrupted by noise. This point is especially important for matrix F as it is to be inversed. The straightforward application of expression (1) leads to an ill-conditioned unfolding matrix. We choose to apply Tikhonov regularization [3] for the stabilisation of F. This technique has already been applied in the context of SENSE reconstruction [4], see [5] and [6] for reference. We perform a Singular Value Decomposition (SVD) of matrix F:

$$F = U\Sigma V^{H} = \sum_{i=1}^{n} \mathbf{u}_i \sigma_i \mathbf{v}_i^{H} ,$$

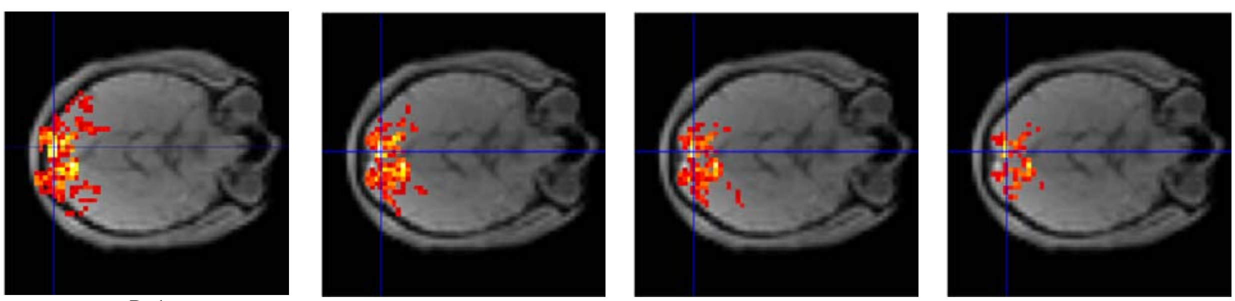
where U = ($\mathbf{u}_1, \mathbf{u}_{2,\dots,}\mathbf{u}_n$) and V = ($\mathbf{v}_1, \mathbf{v}_{2,\dots,}\mathbf{v}_n$) are complex matrices with orthonormal columns, and Σ = diagonal($\sigma_1, \sigma_{2,\dots,}\sigma_n$) is a real matrix containing non-negative diagonal elements appearing in non-increasing order such that $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_n \geq 0$. A stable inverse of F can be calculated in the Tikhonov sense by $F^{-TIK} = V\Sigma^{-TIK}U^{H}$, where Σ^{-TIK} = diagonal($f_1/\sigma_1, f_2/\sigma_{2,\dots,} f_n/\sigma_n$) with $f_i = \sigma_i^2/(\sigma_i^2 + \lambda^2)$. In our implementation, a unique Tikhonov parameter λ is chosen for the whole image. The Tikhonov regularized unfolding matrix becomes $N = R\ F^{-TIK}$.

Materials and Methods

A functional MRI (fMRI) was performed on a healthy volunteer using a 1.5T GE Signa scanner and an 8 channel head coil array. Two EPI sequences at reduction factor 1 and 2 were acquired. The volunteer was instructed to visualize the image of a contrast-reversing chessboard. The experiment was composed of 4 rest/stimulus periods: a rest period consisting of the visualization of a black image during 19.5 seconds and a stimulus period consisting of the visualization of the flashing chessboard during 19.5 seconds. We used a 64x64 sum-of-squares image and eight 64x32 folded images to learn the per pixel unfolding operator. These training images were the result of averaging all images in two short fMRI series acquired without stimuli at reduction factor 1 and 2. This aims to decrease the effect of noise. For the detection of functional activations, reconstructed images at reduction factor 1 and 2 were processed with SPM2 (Statistical Parameter Mapping software, version II), [7].

Results

In Figure 1 we show activation maps calculated by use of SPM2. The activations on the left hand side image of the figure are extracted from a reference series at reduction factor 1 (R=1), the rest of the maps are extracted from R=2 series reconstructed by PreLearn, using different Tikhonov parameters. The cross between the blue lines indicates the voxel presenting the stronger activation on the map. In Figure 2 we draw a curve showing the number of detected activated voxels (Y axis) against different regularization parameters (X axis, note that units in this axis are divided by 1 000).



R=1 | R=2, λ^2=45 000 | R=2, λ^2=10 000 | R=2, λ^2=5 000

Figure 1. Activation maps extracted from fMRI series using reduction factor 1 (R=1, left image) and 2 (R=2, rest of the images). R=2 series are reconstructed using PreLearn with different Tikhonov parameters (λ).

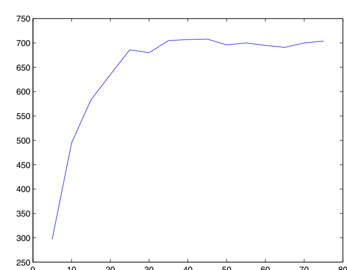


Figure 2. Detected activated voxels versus Tikhonov parameter

Discussion

We see in Figure 1 that for different parameters we obtain different number of activations. Moreover, from Figure 2 the effect of the Tikhonov parameter appears clearly: after an initial interval of suboptimal values it reaches a plateau where the optimal parameter can be considered any value. The exact maximum in our example is λ^2=45 000, where 707 activated voxels are found while 1031 are detected at R=1. We note that the optimal parameter for the detection of the activations is not necessarily the one that obtains the best visual appearance, however, both properties can be reached simultaneously due to the large choice of parameters in the plateau.

Conclusion

We evaluated the effect of Tikhonov regularization on the detection of activations in fMRI using PreLearn. These presented results show that an important part of the activations found at reduction factor 1 are preserved when the optimal regularization parameter is chosen.

References: [1] Ribés et al. ISMRM05, 2005, 472. [2] Ribés et al. ESMRMB05, 2005, 189. [3] Tikhonov, Arsenin. *Solution of ill-posed problems.* 1977. [4] Pruessmann et al. MRM, 1999, 42:952-962. [5] King, Angelos. ISMRM, 2001, 1771. [6] Lin et al. MRM 2004;51:559-567. [7] http://www.fil.ion.ucl.ac.uk/spm/