Point Spread Functions in k-space and image-based Parallel Image Reconstructions

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Introduction: The performance of a given parallel image reconstruction is generally quantified using metrics such as the *g*-factor, which measures the degree of noise amplification in reconstructed images. Noise amplification can be reduced by a variety of conditioning and regularization techniques. In some reconstructions the role played by regularization is explicit; in other cases, such as SMASH (SiMultaneous Acquisition of Spatial Harmonics) and related reconstructions that are theoretically approximate (1), the conditioning of the reconstruction is largely implicit in the approximation made in the reconstruction. Regularization (2, 3) generally reduces noise at the cost of increased artifact. To date, tools for quantifying the tradeoffs between noise amplification and artifact power have been lacking. Here we propose the use of explicit point-spread functions (PSFs) as a tool for assessing residual artifact. As an illustration of this technique we compute the PSFs for two image reconstructions implementing a GRAPPA-like (Generalized Auto-calibrating Partially Parallel Acquisition, 4) reconstruction that involves an inherent approximation in the reconstruction and a theoretically exact SENSE-like (Sensitivity Encoding, 5, 6) technique.

Theory: The point-spread function $PSF_j(x)$ is defined as the function that yields the intensity of pixel j upon convolution with the magnetization density $\rho(x)$. In standard Fourier reconstructions, the PSF is translationally invariant and depends only on the difference $(x_j - x)$. In parallel imaging, the use of coil sensitivity information breaks this translational invariance, and so we explicitly separate the roles of the pixel index j and the spatial coordinate x.

PSFs for SMASH like reconstructions: In SMASH-like reconstructions, including GRAPPA, missing lines in *k*-space are filled in using linear combinations of adjacent lines prior to taking the Fourier transform to obtain an image. In GRAPPA, individual component coil images are reconstructed prior to forming a final sum-of-squares image. Explicitly, this process takes the form

$$\rho_{l,j} = \sum_k e^{ikx_j} s^{full}_{l,k} = \sum_k e^{ikx_j} \sum_{k'l'} T_{kl,k'l'} s_{l'k'} = \int dx' \left(\sum_{kk'l'} e^{ikx_j} T_{kl,k'l'} e^{-ik'x'} C_{l'}(x') \right) \rho(x') \equiv \int dx PSF_{l,j}(x') \rho(x'),$$

where $T_{kl,k'l'}$ is a matrix of weights that converts the under-sampled signals s_{kl} at *k*-space location *k* in coil *l* into the fully sampled data s^{full}, and $C_l(x)$ is the sensitivity of coil *l*, In the last step, we identify the quantity in parentheses as the PSF for coil *l*. Because GRAPPA is a sum-of-squares reconstruction, it does not have a PSF that enters linearly into the final image intensity. In this case, the sum-of-squares combination of the component coil PSFs provides a measure of artifact power.

PSFs for SENSE-type reconstructions: Encoding matrix reconstructions can be formulated in terms of a linear inverse problem of the form $s=B\rho$, where s is a signal vector and B is the encoding matrix. The spin density is reconstructed by applying a suitable (pseudo) inverse of the encoding matrix to s. The PSF can therefore be found from the expression

$$\rho_j = \sum_k B^{-1}_{j,kl} s_{kl} = \int dx' \left(\sum_{kl} e^{-ikx'} B^{-1}_{j,kl} C_l(x') \right) \rho(x') \equiv \int dx' PSF_j(x') \rho(x') \; .$$

In this case, the reconstruction has a well-defined linear PSF, and no sum-of-squares combination is needed.

Results: A 2D image of a spherical phantom was acquired fully-sampled with a matrix of 128×128 using an 8-channel phased-array head coil from which *k*-space was under-sampled to mimic accelerated acquisition with an outer reduction factor (ORF) of 4 and with a dense center of 32 fully sampled lines. In-house implementations of a GRAPPA and a SENSE reconstruction were applied to the data with the matrix of weights $T_{kl,k'l'}$ and the encoding matrix $B_{j,kl}$ used to compute the PSF for the central line through the image. Pure coil sensitivities $C_l(x')$ were estimated by normalizing the individual coil images to the root-sum-of-squares combination, and were zero-padded to 8× resolution prior to FFT to enable the final PSF to be calculated for that higher resolution. Figure 1 shows the appearance of residual artifacts in the magnitude images resulting from GRAPPA and SENSE reconstructions. The PSFs for each are shown in Figure 2.

Discussion: Features in the PSF are identifiable in the images in Fig. 1, for example, greater ringing is seen in the SENSE image corresponding to the well-structured *sinc*-function character of the PSF, whereas the wider central peak in the GRAPPA PSF relates to the smoother appearance of the phantom in the image and may also have an influence on the noise in the image. The peaks at ¼ field-of-view positions, corresponding to the ORF of 4, are more pronounced in the PSF for GRAPPA than for SENSE and manifest as the edge artifacts which are more prominent in the GRAPPA image. Important metrics of the PSF may include the relative power or width of the central lobe, and the relative magnitudes of the side lobes at fractional fields of view. Calculation of the PSFs for any given reconstruction technique as demonstrated here is important in the assessment of a parallel imaging technique in addition to the familiar *g*-factor noise amplification.

References: 1. Sodickson. MRM 2000 44:243-51. 2. Lin et al. MRM 2004 51:559-67. 3. Qu et al. JMRI 2006 244:248-55. 4. Griswold et al. MRM 2002 47:1202-10. 5. Pruessmann et al. MRM 1999 42:952-62. 6. Sodickson et al. Med Phys 2001 28:1629-43

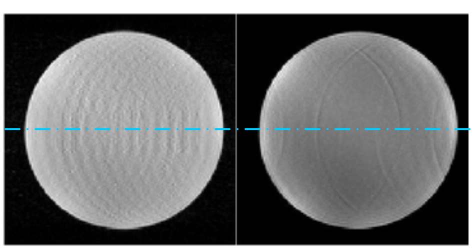

Figure 1: Magnitude images from SENSE (left) and GRAPPA (right) reconstruction techniques of the same *k*-space. Differing degrees of ringing, smoothness and edge artifact are evident. PSF calculated along blue line (Fig. 2).

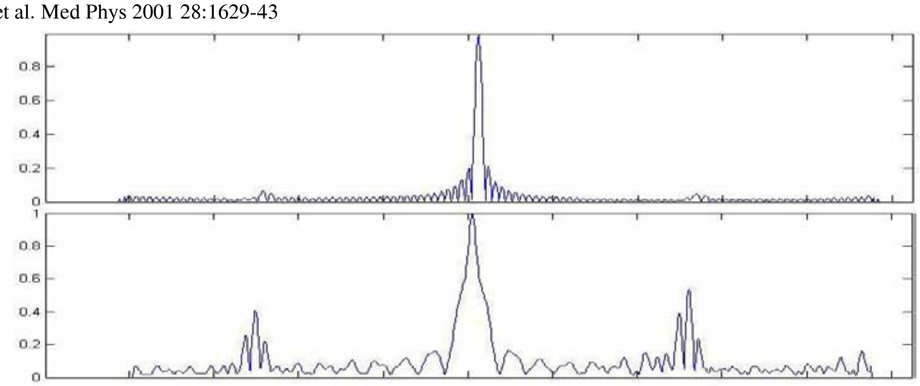

Figure 2: PSFs calculated for the SENSE (top) and GRAPPA (bottom) images shown in Fig. 1. Magnitude is plotted against *x*-position along a line through the center of the phantom (shown blue).