Universal Approach to Quantification of SNR and g-Factor for parallel MRI

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Introduction: The analysis of Signal-to-Noise-Ratio (SNR) in images acquired with accelerated parallel Magnetic Resonance Imaging approaches is frustrated by the fact that image noise becomes spatially-variant precluding the conventional Region-of-Interest (ROI) approach which estimates image SNR from a region of signal within the object and from a region of noise outside of the object (1). Analytical approaches exist (2, 3) for determining the image noise and the *g*-factor which have been shown (4) to allow the production of fully reconstructed images in units of SNR to complement the conventional magnitude image. However, these methods can not be applied when the reconstruction technique used does not explicitly yield the weighting factors required to calculate image noise and *g*-factor, for example, when iterative conjugate-gradient (CG) reconstruction algorithms are used (5) with Sensitivity Encoding (SENSE) (2) or Generalized Encoding Matrix (GEM) reconstruction (6). Furthermore, some parallel imaging techniques such as Generalized Auto-Calibrating Partially Parallel Acquisition (GRAPPA) (7) do generate weights for intermediate images, but then combine those images with a non-linear operation to form the final image, thus making simple analytical SNR calculation for the final image impossible. A universal method for quantification of image SNR and *g*-factor resulting from all parallel imaging techniques will permit objective comparison between different *k*-space sampling schemes, trajectories, or image reconstruction methods.

Methods: The *pseudo multiple-replica SNR measurement* outlined here is equivalent to the gold-standard approach of determining image SNR from the pixel-by-pixel evaluation of image signal mean and standard deviation through a stack of separately acquired images. Actual noise received by each element in the phased-array coil is measured once during a *"noise pre-scan"* by opening the receiver with no applied RF-pulses and with the coils loaded by the object as for imaging. From these data the noise covariance matrix may be formulated (4). Undersampled *k*-Space data is acquired only once, but reconstructed repeatedly with added synthetic random normally distributed noise added before reconstruction of each replica image. That noise has been correlated and scaled by the square-root (Cholesky decomposition) of the noise covariance matrix before addition to the acquired *k*-space.

The *pseudo multiple-replica SNR measurement* was validated in simple phantom images by comparison with gold-standard multiple-replica SNR measurements and the analytical image noise method (2) using an iterative CG-GEM reconstruction with a *k*-space sampling scheme for which analytical matrix-inversion was also possible. Gold-standard *g*-factor maps are the ratio of the image standard deviations (SD) through the stack of replicas for an accelerated and an un-accelerated image reconstruction, divided by $\sqrt{R}$ (acceleration factor, R). *g*-Factor maps were produced for the *pseudo multiple replica* CG-GEM-reconstruction from the noise SD of an accelerated image and from a fully sampled *pseudo*-image, where the latter is reconstructed from an entirely synthetic noisy *k*-space with no signal present; it would not be feasible to acquire a separate fully sampled *k*-space for *in vivo* studies. Finally this approach was applied to a 3D examination of the abdomen *in vivo* with 2D-acceleration.

Results: Figure 1 shows SNR maps (top) of a 2D slice through a phantom (matrix 64 × 64) reconstructed from *k*-space data decimated by a factor of 4 with a fully sampled center of 16 lines giving an acceleration factor $R = 2.3$. There is excellent agreement between the SNR maps calculated with the analytical method, the *pseudo multiple-replica* method, and the actual multiple-replica method both in their spatial distributions and in the overall scaling of their SNR values. The accompanying *g*-factor maps (bottom) show that areas of high *g*-factor correspond well to regions of low SNR and also show agreement between all methods. Figure 2 shows *in vivo* SNR and *g*-factor maps for a 3D coronal slab abdominal scan (matrix 256 × 110 × 16, read-out SI) accelerated 3.6-fold (with reduction factors of 3.0 Left-Right and 1.2 Anterior-Posterior) using an 8-element phased-array coil. The *in vivo g*-factor map, which is impossible to obtain without use of the *pseudo multiple-replica* method, shows spatially-variant noise amplification for this accelerated image acquisition.

Discussion: Synthesis of correctly scaled and correlated noise allows this method to faithfully emulate the gold-standard multiple replica SNR measurement and allows this method to be universally applied with any parallel imaging technique. The noise pre-scan is both easy and rapid, sufficient data may be acquired in seconds or less in a few additional read-outs, allowing SNR and *g*-factor to be measured for every *in vivo* image acquisition without significantly lengthening either breath-hold duration or total exam time. SNR-unit images and *g*-factor maps shown here for a CG-GEM-reconstruction may also be produced for other parallel imaging techniques provided the image-noise in each image replica may be formed from either magnitude or complex image data. Furthermore, the *pseudo*-technique is likely to out-perform actual multiple replica by eliminating the influence of instrumental drift.

Conclusion: The *pseudo multiple-replica SNR measurement* outlined will provide a useful tool for objective comparison between the *in vivo*-performance of any parallel imaging acquisition scheme for clinical imaging protocols which use reconstructions that do not allow direct calculation of *g*-factor.

References: 1. Reeder et al. MRM 2005 54:748-54. 2. Pruessmann et al. MRM 1999 42:952-62. 3. Sodickson et al. MRM 1999 41:1009-22. 4. Kellman et al. MRM 2005 54:1439-47. 5. Pruessmann et al. MRM 2001 46:638-51. 6. Sodickson et al. Med. Phys. 2001 28:1629-43. 7. Griswold et al. MRM 2002 47:1202-10.

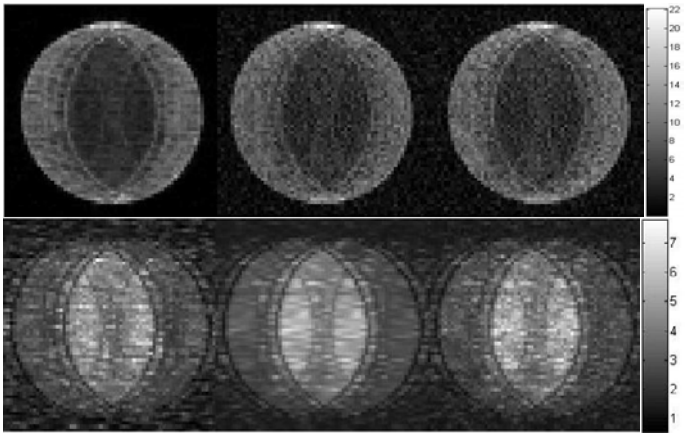

Figure 1: SNR maps in a phantom obtained from gold-standard multiple-replica assessment (top-left), analytical image-noise calculation (top-middle), *pseudo multiple-replica* (top-right) with gray-scale from 0 to 22; corresponding *g*-factor maps (bottom), grayscale from 1 to 7.5.

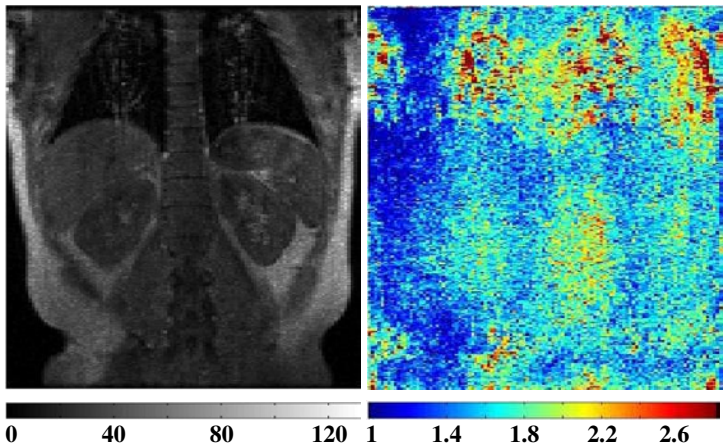

Figure 2: SNR (left) and *g*-factor (right) maps from an *in vivo* T_1-weighted abdominal scan using the *pseudo multiple-replica* method; in this image plane acceleration is Left-Right.