SNR Variation with Regularization Term for Non-Cartesian SENSE Reconstruction

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Introduction

Reconstructing an image from k-space data using the SENSE parallel imaging algorithm [1] involves solving an ill-conditioned inverse problem, which in general requires some form of regularization. Standard Tikhonov regularization introduces a scalar free-parameter (damping term) δ into the reconstruction that has the effect of suppressing small singular values in the sensitivity matrix, which reduces their influence on the computed solution. It is important to recognize the effects of the regularization parameter on the SNR characteristics of the resulting images, and this must be considered if SNR is compared between different speed-up factors. For example, in previous functional MRI studies, the temporal SNR has been found to deviate from the expected $\sqrt{R}$ decrease with speed-up R [2,3,4,5]. In this study, the effect of the regularization parameter on the SNR of reconstructed spiral images is investigated using SENSE and a speed-up factor 2. Phantom data are presented; similar results are obtained *in vivo* using the same acquisition protocol.

Methods

Data were acquired on a GE Signa Excite 3 Tesla whole body system equipped with an 8 channel receive-only head coil and body coil for RF transmission. Single-shot images of a homogeneous phantom were acquired using a spiral readout; the readout window was shortened by reducing the sampling density in the radial direction to obtain a speed-up factor of 2. In order to create coil sensitivity profiles for image reconstruction, reference images in which k-space was fully sampled (2 interleaves) were acquired and smoothed by Gaussian convolution.

All image reconstruction was performed off-line using the SENSE iterative approach designed for arbitrary k-space trajectories [6]. Our MATLAB implementation used a Kaiser-Bessel gridding kernel of width 5, oversampling factor 2 and shape α=11.7. As described in [7], the regularization term δ is specified as a fraction (10^{-6} to 10^{-1}) of the maximum singular value and exhibits a spatial dependence given by the inverse of the signal amplitude [8]. Density weighting and intensity correction were used to improve convergence and the number of iterations was 30.

The geometry factor, g, is a spatially variant noise amplification parameter that relates to the k-space sampling strategy and coil geometry, and is ideally equal to one. Estimates of g were made by replacing the k-space data with complex noise in 200 'noise' data sets and calculating the pixel-wise standard deviations of the resulting images, then g is the ratio of the std maps for R=2 and R=1:

$$\sqrt{2}g = \frac{std(Noise\ images_{R=2})}{std(Noise\ images_{R=1})} \qquad (1)$$

Results

Figure 1 shows reconstructed images for five regularization parameters. The regularization suppresses spiral aliasing artifacts and the overall noise, which suggests a moderate level of regularization is optimal in terms of image quality.

Figure 2 shows 'noise' maps generated from reconstructing 200 noise only data sets and taking the pixel-wise standard deviation of these images. The noise maps are mostly uniform indicating a benign g-factor, which is expected for this trajectory and coil geometry.

Figure 3 shows the mean signal inside the phantom images (R = 2) and the standard deviation of the 'noise' maps (R = 1 and 2). As expected, the standard deviation decreases as the regularization term increases; the results indicate a rapid initial drop-off followed by a slow decrease. By observing the images in Figure 1 the initial drop-off corresponds with a reduction in spiral aliasing artifacts in the center of the phantom and the slower decrease corresponds with a reduction in the overall noise. Figure 4 shows the ratios of the standard deviations given by Eq 1. When the regularization term is greater than 0.001, the g factor becomes <1, and so the expected $\sqrt{2}$ decrease in SNR when using speedup factor 2 is not seen.

Figure 1: Images (R=2) obtained for different regularization parameters.

δ 10^{-6} 0.001 0.005 0.020 0.100

Figure 2: Noise maps (R=2) for different regularization parameters.

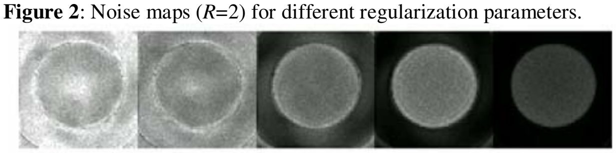


Figure 3. Mean signal for R=2(black) and standard deviation for R=1 and 2. The values are taken over the phantom for each regularization term.

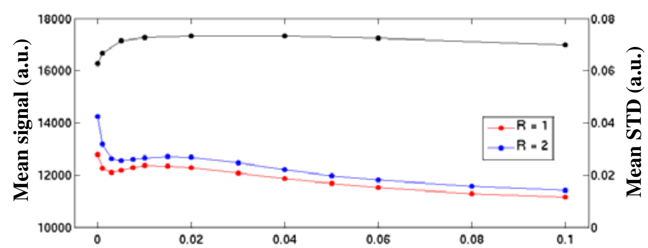
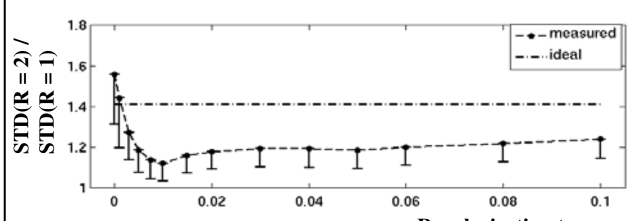


Figure 4. Ratio of standard deviations for R=2 and 1 (from Fig 3). 'Ideal' line represents g=1 (i.e. $\sqrt{2}$). Points below the line represent g<1.



Discussion

SENSE reconstruction solves an ill-conditioned inverse problem and in general requires regularization to obtain acceptable image quality. The SNR is highly dependent upon the specific regularization parameter used [9]. Previous studies have reported SNR changes with parallel imaging speed-up factor that differ from the expected square root relationship.

The present study has observed the same phenomenon using SENSE and a Tikhonov regularization strategy that suppresses signals in inverse proportion to their amplitude [7]–[8]. The type of regularization used in parallel imaging reconstruction is critical when comparing SNR measurements at different speed-up factors.

Different parallel imaging reconstruction methods use different types of processing that may regularize the inverse problem in ways that are not easily comparable with the present approach.

References

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