

# Iterative Back-Projection Reconstruction For Radial SENSE

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## Introduction

Reconstruction of Sensitivity encoded MRI with non-Cartesian trajectories involves inverting very large generalized encoding matrix (GEM) which is computational demanding. In this work, a novel efficient approach for radial SENSE reconstruction is introduced which combines the classic back-projection technique with conjugate-gradient (CG) iteration. Taking advantage of the projection-slice theorem, the GEM can be significantly sparsified by FFT. In each iteration, the matrix-vector multiplications are performed by projection and back-projection, eliminating the involvement of gridding process.

## Method

According to the projection-slice theorem, the projection of an object taken at angle  $\phi$  is related to a radial line with angle  $\phi$  in k-space by 1DFFT. The reconstruction can then be formulated as a new linear system:  $p=Rm$ , where  $p$  is alignment of the image-domain projection data from all the channels obtained by applying 1DFFT to the radial MR data;  $R$  is the encoding matrix which is composed of sensitivity encoding and projection encoding. In analogy to the gridding-based iterative reconstruction method for generalized SENSE [1,2], CG iteration is applied to the normal equation form:  $R^H R m = R^H p$ . The operator  $R$  can be applied by two steps: (1) the image  $m$  is masked by RF sensitivities of each coil individually to simulate sub-images of different channels; (2) for each channel Radon transform is applied and the resultant projection data of all the channels are aligned in a vector. In a similar way,  $R^H$  can also be applied by two steps: (1) direct back-project (without filtering) the projection data onto the image pixels in each individual channel (it is easy to see the conjugate transpose of radon transform operator is exactly the direct back-projection); (2) the sub-images resulted from back-projections are masked by the complex conjugate of the RF sensitivities, and then summed up to combine a single image. The entire process of back-projection based iterative reconstruction is illustrated in Fig. 1.

Parallel radial imaging with a Shepp-Logan phantom was simulated. For  $64 \times 64$  imaging, 16 projections were acquired with 80 samples in each view using 8 receiving channels. Axial brain MRI data were acquired using a Siemens true-FISP radial sequence with an 8-element head coil array on 1.5T system. For  $128 \times 128$  imaging, a total of 32 projections were acquired with 128 samples in each projection.

## Results

Simulated radial images with iterative back-projection reconstruction are shown in Fig. 2. The reconstructions converge very fast with iterations. With 16 projections by 8 RF channels, good image quality can be achieved in less than 10 iteration loops. Computation time for each iteration loop was around 0.08 second, even more efficient than the gridding-based reconstruction for this low-resolution case.

In vivo results are shown in Fig. 3. With 32 projections by 8 RF channels, relatively artifact-free images with matrix size  $128 \times 128$  can be reconstructed in 7 iteration cycles. Each iteration took around 0.8 second, longer than the time cost of the gridding-based counterpart (around 0.7s) for this case.

In both cases, in analogy to gridding-based CG method, noise amplification effects appeared after several tens of iteration loops. This can be addressed by a proper stop criterion or by applying inner-regularization technique [3,4].

## Conclusion

Conventional back-projection technique is extended for reconstructing sensitivity-encoded radial MRI. The generalized encoding matrix (GEM) is inverted by conjugate-gradient (CG) iteration, with matrix-vector multiplication performed by projection and back-projection. The feasibility of this novel method is demonstrated by simulation and MRI experiments.

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**References** [1]. Pruessmann KP, et al. MRM 46: 638-651 (2001)  
[3]. Qu P, et al. MRM 54: 1040-1045 (2005)

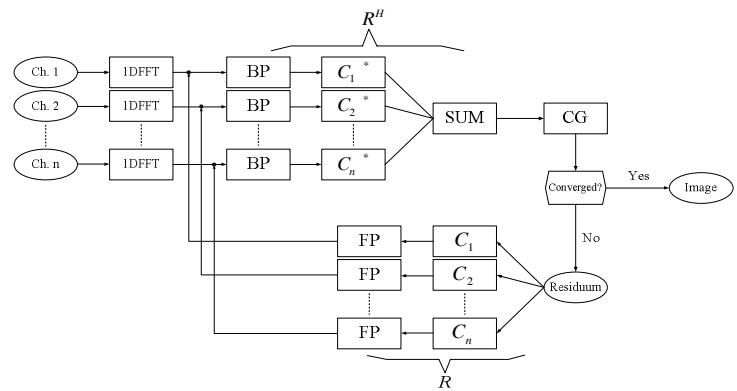


Fig. 1. Schematics of iterative back-projection radial SENSE.

BP: back-projection FP: forward projection.

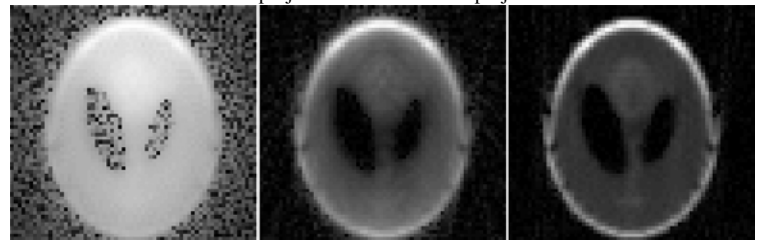


Fig. 2. Progression of iterative back-projection reconstruction of simulated  $64 \times 64$  phantom images after 1, 3, 5 iterations, respectively.

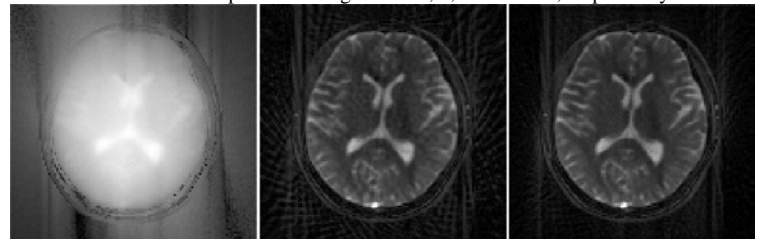


Fig. 3. Progression of iterative back-projection reconstruction of  $128 \times 128$  brain images after 1, 3, 7 iterations, respectively.

[2]. Zhang B, et al. Proc ISMRM 2004. p450.

[4]. Sánchez-González J, et al, MRM 55: 287-295 (2006)