Continuous 2D GRAPPA kernel for propeller trajectories

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Introduction: For Cartesian acquisitions, a single 2D GRAPPA kernel is typically used to synthesize the missing data in an accelerated scan. For undersampled non-Cartesian trajectories like e.g. radial, spiral or PROPELLER, the local neighborhood varies over k-space. This requires the calculation of a new 2D GRAPPA kernel for some (or even every) undersampled k-space location. The latter is the essence of the PARS technique (1). This multitude of estimations is very time consuming in the general case. For undersampled propeller trajectories (such as PROPELLER (2), Turbo-PROP (3) and SAP-EPI (4)), the GRAPPA estimation becomes simpler since the undersampled trajectory is Cartesian and equidistant within each blade. Across blades, the GRAPPA kernel has the same spacing between the acquired and missing points, but a unique GRAPPA kernel must be calculated for each blade due to the orientation dependence relative to the RF coil. Given some N fully sampled calibration blades, two concerns may be raised for GRAPPA combined with propeller trajectories. First, the amount of data in a (low resolution) calibration blade is typically less than for conventional Cartesian GRAPPA acquisitions (e.g. FSE). This may render the problem underdetermined for large GRAPPA kernel sizes and many-(?)coil arrays. Second, given the slow and continuous variation of the coil sensitivities, it is unlikely that the 2D GRAPPA kernel would differ much from one blade to another. For these reasons, we have in this work investigated the use of a continuous 2D GRAPPA kernel tailored for propeller trajectories.

Materials & Methods: Building on the ideas from our earlier work on spatial modeling of 1D GRAPPA kernels in hybrid space (5), we have now used a similar continuous representation of the GRAPPA weights using a 2D GRAPPA kernel in k-space with the continuous representation being the angle of the blades. The weight estimation for blade 1, in which the relation between ACS line i for coil j, $\mathbf{y}_{ij}$, and the surrounding acquired lines (all coils), contained in a matrix $\mathbf{A}$, is given by $\mathbf{y}_{ij,1} = \mathbf{A}_1\mathbf{w}_{ij,1}$, where $\mathbf{w}_{ij,1}$ is an $(N_{coils}\times N_{ky}\times N_{kx})\times 1$ vector representation of the GRAPPA kernel, and N_{ky} and N_{kx} are the number of rows and columns involved in the GRAPPA kernel. In our setting $\mathbf{w}_{ij}$ is of size $(8\times 2\times 3)\times 1$. We can now estimate the GRAPPA weights for all N blades at once by:

$$\mathbf{y}_{big} = \begin{bmatrix} \mathbf{y}_{ij,1} \\ \vdots \\ \mathbf{y}_{ij,N} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_N \end{bmatrix} \begin{bmatrix} \mathbf{w}_{ij,1} \\ \vdots \\ \mathbf{w}_{ij,N} \end{bmatrix} = \mathbf{A}_{big}\ \mathrm{vec}(\mathbf{W}_{ij}) \quad [1]$$

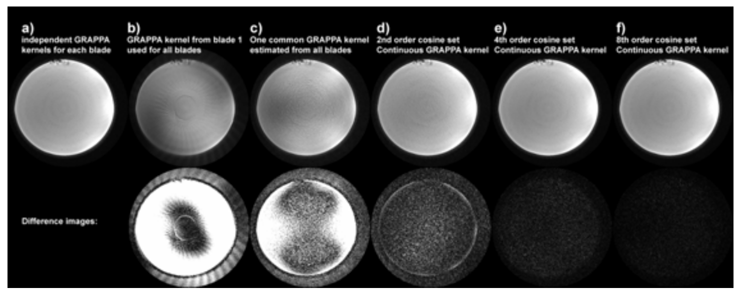

Figure 1. Different GRAPPA kernels used to synthesize missing lines in 20 propeller blades, each undersampled by R=2. See above and text for details. A 4th to 8th order cosine set can accurately model the variations in GRAPPA coefficients over a 180° sweep

where $\mathbf{W}$ is a matrix of size $(N_{coils}\times N_{ky}\times N_{kx})\times N$ and the "vec" operator unravels the matrix $\mathbf{W}$ into a single column. To reduce the total number of independent unknowns in $\mathbf{W}$ and to make the GRAPPA kernel a continuous function over the blade angles, $\mathbf{W}$ is constrained by a cosine basis set $\mathbf{C}$ of size $N\times N_{order}$ across the columns of $\mathbf{W}$: $\mathbf{W} = (\mathbf{CH})^T$. Given the basis set $\mathbf{C}$, $\mathbf{W}$ (and hence the GRAPPA coefficients for all blades) is solely determined by the unknown elements of $\mathbf{H}$, which is arbitrarily smaller than $\mathbf{W}$. The elements of $\mathbf{H}$ are estimated from the k-space data $\mathbf{A}_{big}$ and $\mathbf{y}_{big}$ via

$$\mathbf{y}_{big} = \mathbf{A}_{big}\ \mathrm{vec}\left(\mathbf{H}^T\mathbf{C}^T\right) = \mathbf{A}_{big}\underbrace{\left(\mathbf{C}\otimes\mathbf{I}\right)}_{\mathbf{Q}}\underbrace{\mathrm{vec}\left(\mathbf{H}^T\right)}_{\mathbf{h}} = \left(\mathbf{A}_{big}\mathbf{Q}\right)\mathbf{h} \ \Rightarrow\ \mathbf{h} = \left(\mathbf{A}_{big}\mathbf{Q}\right)^+\mathbf{y}_{big} \quad [2]$$

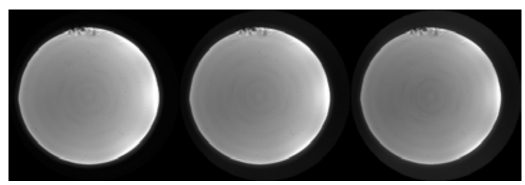

Figure 2. Reconstructions of 100 propeller blades undersampled by a factor of R=2. An 8th order continuous 2D GRAPPA kernel has been derived from all blades (left), every third blade (middle), every 9th blade (right)

where "+" denotes the pseudo inverse and "$\otimes$" the Kronecker product and we have used the following algebraic rule: $\mathrm{vec}(\mathbf{XY}) = \left(\mathbf{Y}^T\otimes\mathbf{I}\right)\mathrm{vec}(\mathbf{X})$. Interestingly, the number of rows of $\mathbf{C}$ may be different when estimating and applying $\mathbf{h}$. This allows e.g. the use of a sub-set of the final blades for the calibration. Exactly how many blades required for the estimation of $\mathbf{h}$ depends on the 2D GRAPPA kernel size, number of coils used and the richness of the basis set. To evaluate this method, we acquired propeller data on a gel phantom using an 8-channel head coil on a GE 1.5T system. A SAP-EPI pulse sequence used with 100 blades of resolution 32×256 swept over 0-180°, with R=2 along the long-axis of the blade. Two reconstruction experiments were performed. First, we investigated how many basis functions are needed to properly model the angular change of the GRAPPA kernel over the acquired blades for our coil configuration. For this experiment, every fifth blade was used to mimic a clinical scan situation better using 20 blades. Second, we explored how many blades are needed for a proper estimation of $\mathbf{h}$ and how well the continuous GRAPPA kernel performs when applied to all 100 blades in the application phase.

Results: Figure 1 shows reconstructions of 20 equidistant blades with R=2 using various GRAPPA kernels. Fig. 1a corresponds to independent GRAPPA kernels derived on a blade per blade basis. In contrast, Fig. 1b shows the effect of reusing the GRAPPA kernel obtained from blade 1 on all 20 blades, which results in significant artifacts. In Fig. 1c, a single GRAPPA kernel has been derived from all blades using Eq. 2 in which case $\mathbf{C}$ is a single column containing N ones. This trades the aliasing seen in Fig. 1b for additional noise - again demonstrating that the GRAPPA kernel must be varied over angles. However, with only a few basis functions, the situation is much improved. In Fig 1d-f, two, four and eight cosine basis functions have been used for $\mathbf{C}$ in Eq. 2. Comparing Fig. 1a and 1e, we have reduced the number of GRAPPA coefficients to be determined by a factor of 5 without noticeable artifacts. With more blades, the reduction of unknowns will be even more pronounced. In Figure 2, all 100 blades have been involved in the reconstruction using an 8th order continuous GRAPPA kernel derived from all (left), every third (middle) and every ninth (right) blade. Using only every 9th blade (11%) for the estimation of $\mathbf{h}$ produces still proper reconstructions, although slight ringing may be appreciated in the central region of the phantom.

Discussion & Conclusion: In this study we have demonstrated a way to estimate a continuous representation of the 2D GRAPPA kernel over angles in k-space. For GRAPPA propeller trajectories used in PROPELLER, Turbo-PROP and SAP-EPI this method makes better use of the blade calibration data and provides a flexible trade-off between computational speed and how fast the GRAPPA weight functions are allowed to vary angularly. Moreover, it is possible to estimate the continuous 2D GRAPPA kernel from a subset of the blades and apply the GRAPPA kernel for all blades during a GRAPPA accelerated acquisition. This reduces the number of fully sampled blades needed for the GRAPPA calibration. Extensions of this method may be useful for other non-Cartesian trajectories, like radial sampling, provided the local distances in addition to the angle are taken into account. Indeed, modeling of the angular variation of the GRAPPA weights may have important implications for reconstructions like PARS. Several local angle neighborhood weights, such as used in PARS, may determine a continuous set of GRAPPA weights that may the quickly calculated and applied per radial line, improving PARS reconstructions.

References: 1) Yeh EN *et al.* Magn Reson Med 2005;53(6):1383-1392. 2) Pipe JG. Magn Reson Med 1999;42(5):963-969. 3) Pipe J. 2002; Honolulu. ISMRM p 425. 4) Skare S *et al.* Magn Reson Med 2006;55(6):1298-1307. 5) Skare S, Bammer R. 2005; Miami. ISMRM p 2424.

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