Analytical Sampling Density Compensation for PROPELLER MRI

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Introduction

In PROPELLER MRI [1], the partial overlap of the blades results in a non-uniform sampling density, which has to be compensated by appropriate weighting of the data. In general, the calculation of sampling weights is based on elaborated fast convolution algorithms [2,3], which are computationally efficient, but quite complex to implement.

In the present study, an alternative approach is presented, which takes benefit of the regular-grid properties of the blades to analytically derive the sampling density function as defined by Jackson et al. [3].

Methods

Theory: The sampling density function $P(\mathbf{k})$ as introduced in [3] is defined as the k-space convolution of the sampling function $S(\mathbf{k})$ and the employed interpolation kernel $C(\mathbf{k})$, which corresponds to a multiplication in image space (Eq. 1). The sampling function of a single blade represents a regular, finite grid, whose Fourier transform $s_{\text{blade}}(x,y)$ may be written analytically as an grating lobe function (Eq. 2), and hence, the overall sampling function $s(x,y)$ may be obtained by superposition (Eq. 3). Here, N_x, N_y, B and ϕ denote the number of samples per line, the number of lines per blade, the number of blades and the blade rotation angle, respectively. Therefore, $s(\mathbf{r})$ can simply be calculated for arbitrary image locations, and subsequently, $\rho(\mathbf{r})$ may be obtained by multiplication with $c(\mathbf{r})$, the Fourier transform of the interpolation kernel (cf. Eq. 1). Finally, $P(\mathbf{k})$ of a particular, rotated blade may be derived by discrete Fourier transform of $\rho(\mathbf{r})$, sampled at the accordingly rotated image grid.

$$\mathrm{P}(k) = S(k) * C(k) \overset{FT}{\Leftrightarrow} \rho(r) = s(r)\cdot c(r) \qquad [1]$$

$$s_{blade}(x,y) = \sum_{\forall l,m \in grid} \exp(i2\pi(lx+my)) = \frac{\sin(2\pi N_x x)\sin(2\pi N_y y)}{\sin(2\pi x)\sin(2\pi y)} \qquad [2]$$

$$s(x,y) = \sum_{k=1..B} s_{blade}(x\cos(\varphi_k) - y\sin(\varphi_k), x\sin(\varphi_k) + y\cos(\varphi k_k)) \qquad [3]$$

Implementation: The proposed approach has been implemented in C using a processor-optimized FFT-library [4]. For numerical efficiency, the grating lobe and kernel functions were pre-calculated and stored as function tables. $P(\mathbf{k})$ was calculated on a $N_x \times N_x$ k-space grid coinciding with one of the blade grids. For a PROPELLER trajectory with B-fold symmetry, $P(\mathbf{k})$ is identical for all blades, which results in a computational complexity of $O(\mathrm{N} \cdot \mathrm{B})$, where $\mathrm{N} = N_x^2$ denotes the number of image pixels.

Simulations: PROPELLER k-space datasets were synthesized analytically using the Shepp-Logan phantom [5]. For image reconstruction, the data were weighted according to the proposed approach and rotated to a common Cartesian grid by discrete sinc-interpolation [6]. Additional reconstructions with all data set to "1" were performed to obtain the principal transfer function $S(\mathbf{k})/P(\mathbf{k})$. For comparison, also Cartesian data sets with same resolution were synthesized and conventionally reconstructed.

Results and Discussion

On a 2GHz Pentium machine, the processing time for sampling density correction was about 70 ms for a symmetric trajectory with N_x =255, N_y= 21 and B=19. The principal transfer function showed only minor modulations at the intersections of the blades, and the mean relative deviation was less than 2%. The image quality of the PROPELLER reconstruction was comparable to those of the Cartesian reference image. Selected results are shown in Fig.2. In general, PROPELLER trajectories may be asymmetric as a result of e.g. rotating-motion correction. In this case, $P(\mathbf{k})$ could be recalculated for every blade, leading to a complexity of $O(N \cdot B^2)$, which, however, becomes unfavorable for large blade numbers. Alternatively, $P(\mathbf{k})$ could be successively rotated to the respective blade coordinate systems by direct sinc-interpolation [6], which would preserve the complexity $O(N \cdot B)$ of the symmetric case.

In conclusion, the proposed approach represents a simple, fast and powerful alternative to gridding-based convolution techniques for PROPELLER sampling density correction. The approach will suffer from increasing computation time only in case of an extremely large number of blades.

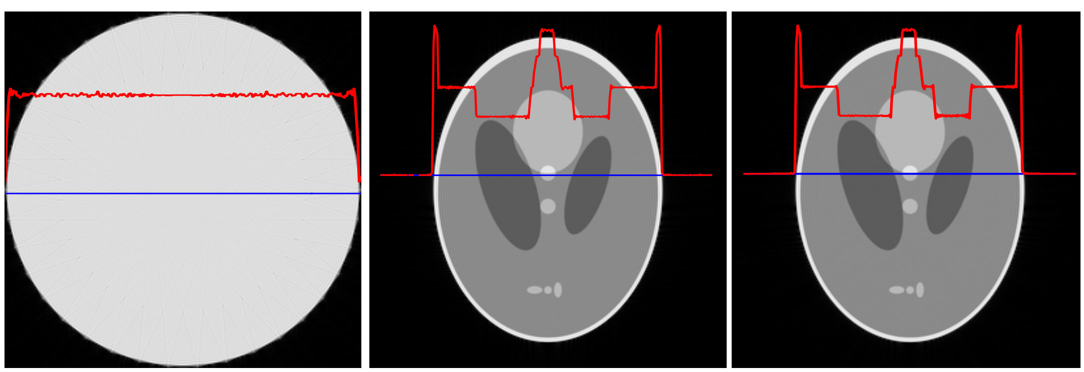

Figure 1: Image quality. The PROPELLER principal transfer function (left) and reconstructed images of the Shepp-Logan phantom (middle: PROPELLER, right: Cartesian) are shown. Additionally, 1D profiles of selected rows (blue lines) are shown as red lines. Using the proposed sampling density compensation very good image quality could be achieved.

References

1. Pipe JG. MRM 1999;42:963-9.
2. O'Sullivan JD. IEEE TMI 1985;4:200.
3. Jackson JI et al. IEEE TMI 1991;10:473
4. Frigo M and Johnson SG. Proc IEEE 2005 93:216
5. Shepp LA and Logan BF. IEEE TNS 1974; 21:21
6. Eddy WF et al. MRM 1996; 36:923