# Variable Slew Rate Spiral Design for Local SAR Reduction in 2D RF Pulse Design 

D. Xu ${ }^{1}$, K. F. King ${ }^{2}$, and Z-P. Liang ${ }^{1}$<br>${ }^{1}$ Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, IL, United States, ${ }^{2}$ General Electric Healthcare, Milwaukee, WI, United States

## INTRODUCTION

Spirals are an important class of trajectories for RF pulse design [1,2]. As suggested by the $k$-space interpretation, 2D spatially selective pulses designed by small-tipangle [1] or linear-class large-tip-angle (LCLTA) [2] methods tend to have high amplitude for the segment that corresponds to near the $k$-space origin, where the amplitude can often exceed specific absorption rate (SAR) or amplifier power limitations, especially when designing large-tip-angle pulses and/or highly-accelerated parallel transmit pulses. Global scaling of the pulse and gradients could reduce the pulse amplitude, but with a huge penalty in pulse duration. Although variable-rate selective excitation [3] has been successfully used to reduce SAR by changing the shape of RF pulse and gradients locally in 1D pulse design with constant gradient, generalizing this method for the 2D case with time-varying gradients has not been shown yet (if possible). The main difficulty is that discontinuity in gradient waveforms would occur when scaling pulse and gradients locally; although the discontinuity can be smoothed by building gradient ramps for the 1D constant gradient case [3], for a 2D spiral, smoothing time-varying gradients locally around the discontinuities in $x$ and $y$ gradients simultaneously while keeping the spiral trajectory unchanged would be difficult (if possible). In this paper, we propose to use variable slew rate spiral trajectories to solve the local SAR overshooting problem.

## PROPOSED METHOD

We consider outward Archimedian spirals in this paper. Inward spirals can be generated by reversing the gradient waveforms of outward spirals. A practical spiral gradient waveform consists of two parts [4,5]: the slew rate limited part and, if the waveform is long enough, the following gradient amplitude limited part. For spiral trajectories used for 2D pulse design, due to the pulse duration constraint, we normally have only limited number of spiral turns, all (or at least the first few turns) of which lie in the slew rate limited part, where the gradient waveform is not limited by the max gradient amplitude but instead by the max gradient slew rate. Therefore, we could indirectly reduce gradient amplitudes locally (and therefore reduce local SAR) by reducing the max allowable slew rate for the corresponding segments while keeping the max slew rate unchanged for the remaining gradients. We call a spiral with such a time-varying max slew rate constraint $S_{\text {max }}(t)\left(S_{\text {max }}(t) \leq S_{\text {max }}^{\circ}\right.$, the hardware limit of slew rate) the variable slew rate spiral. Denoting $\lambda=N_{\text {int }} / D$ as a constant that determines radial sampling interval with $N_{\text {int }}$ being the number of spiral interleaves and $D$ being the field-of-view, $k(\mathrm{t})=k_{x}(t)+i k_{y}(t)$ as $k$-space location at time $t, \theta(t)$ as the azimuthal angle of $k(t)$ in polar coordinates, and $\gamma$ as the gyromagnetic ratio, we can write the variable slew rate spiral mathematically as: $k(t)=\lambda \theta(t) e^{i \theta(t)}$, whose gradient

$$
\begin{equation*}
G(t)=\frac{\lambda}{\gamma} \dot{\theta}(t)(1+i \theta(t)) e^{i \theta(t)} \tag{1}
\end{equation*}
$$

satisfies $S(t) \equiv \dot{G}(t) \leq S_{\text {max }}(t)$. Using similar analysis in [4], a variable slew rate spiral satisfies the following differential equations:

$$
\begin{equation*}
\ddot{\theta}(t)=\frac{f(\theta(t), \dot{\theta}(t))-\theta(t) \dot{\theta}^{2}(t)}{1+\theta^{2}(t)} \tag{2}
\end{equation*}
$$

where

$$
f(\theta(t), \dot{\theta}(t))=\left\{\begin{array}{l}
{\left[\left(\gamma S_{\max }(t) / \lambda\right)^{2}\left(1+\theta^{2}(t)\right)-\dot{\theta}^{4}(t)\left(2+\theta^{2}(t)\right)^{2}\right]^{1 / 2} \text { if }|G(t)|<G_{\max }}  \tag{3}\\
0 \text { otherwise }
\end{array}\right.
$$

with $G_{\max }$ being the max gradient amplitude the hardware can provide. Note $|G(t)|$ is a function of $\theta(t)$ and $\dot{\theta}(t)$ given by Eq. (1), and $S_{\max }(t)$ is given before designing the spiral trajectory. Equations (2) and (3) can be solved by the Runge-Kutta method [6] with boundary conditions $\theta(0)=0$ and $\dot{\theta}(0)=0$. They also permit an approximate analytic solution (similar to [5]), which is faster and more flexible (to adjust parameters). Details of the analytic solution are omitted due to limited space.
RESULTS
We use an example to illustrate the use of variable slew rate spiral in 2D spatially selective refocusing pulse design. The desired flip angle profile after applying the pulse was $180^{\circ}$ inside an infinite cylinder (diameter $=16 \mathrm{~cm}$ ) in the $x-y$ plane and no flip outside ( $D=30 \mathrm{~cm}$ ), as shown in Fig. 1a. The target $k$-space trajectory was a 10 -turn inherently refocused spiral (outward spiral + rephaser, shown in Fig. 1b) [2]. The hardware limits were: $G_{\max }=4 \mathrm{Gauss} / \mathrm{cm}, S_{\max }^{\circ}=14500 \mathrm{Gauss} / \mathrm{cm} / \mathrm{sec}$, and $\max B_{1}=0.25$ Gauss. This spiral trajectory was designed by three methods, each with a different $S_{\max }(t)$. With the above design and hardware specifications, spiral gradient waveforms designed by the three methods contained only the slew rate limited part. As shown in Fig. 1c-e, the constant slew rate spiral $\left(S_{\text {max }}(t)=S_{\text {max }}^{\circ}\right.$, Fig. 1c) produced 3.3 msec gradient waveform and pulse (designed by the LCLTA method [2]), where the pulse had exceedingly high RF amplitude at the beginning of the pulse (max amplitude $=0.69$ Gauss $>\max B_{1}$ ). As a straightforward modification, we reduced $S_{\max }(t)$ from $S_{\max }^{\circ}$ to $\hat{S}_{\max }^{\circ}=14500 / 3^{2}=1611 \mathrm{Gauss} / \mathrm{cm} / \mathrm{sec}($ Fig. 1 f ) to reduce the whole gradient and pulse amplitudes by roughly a factor of three (Figs. 1g-h). Although this global scaling method could reduce the max pulse amplitude ( 0.26 Gauss, which slightly exceeds the 0.25 Gauss limit but very close), it increased the pulse duration significantly ( 9.8 msec ). By using variable slew rate spiral (the numerical solution was adopted for this example), with $S_{\max }(t)$ being a low value in the beginning but high value after the trajectory has moved away from the origin (shown in Fig. 1i), we got a 4.1 msec gradient (shown in Fig. 1j). The resulting pulse is shown in Fig. 1k, whose max amplitude is 0.22 Gauss (< max $B_{1}$ ). Compared to Fig. 1e, only the part of amplitude that exceeded max $B_{1}$ was reduced and the bulk shape/amplitude of the rest of waveform remained unchanged. This localized pulse amplitude reduction is quite efficient and is the main advantage of variable slew rate spirals. The variable slew rate spiral generated a realizable (i.e. satisfying hardware constraints) RF pulse that had comparable duration to the non-realizable pulse ("theoretical" pulse without the max $B_{1}$ constraint, Fig. 1e) and much shorter length than the pulse from global scaling. Reduced relaxation and off-resonance induced image artifacts are expected using the pulse in Fig. 1k compared to the pulse in Fig. 1h.
CONCLUSION
We propose to use variable slew rate spiral trajectories to solve the SAR overshooting problem in RF pulse design. Design examples show that by reducing gradient amplitudes locally, variable slew rate spirals are able to reduce peak RF amplitude without significantly increasing pulse duration.

## REFERENCES

[1] Pauly et al., JM $R$, vol. 81, pp. 43-56, 1989.
[4] King et al., MRM, vol. 34, pp.156-160, 1995.
[2] Pauly et al., $J M R$, vol. 82, pp. 571-587, 1989.
[5] Glover, MRM, vol. 42, pp. 412-415, 1999.
[3] Conolly et al., $J M R$, vol. 78, pp.440-458, 1988.
[6] Kahaner et al., Numerical Methods and Sofware, Prentice-Hall, 1989

