k-Space Sample Density Compensation via Basis Function Cross-Correlations, with Application to the Design of 2D RF Excitations

D. Mitsouras[1], R. V. Mulkern[2,3], O. Afacan[4], D. H. Brooks[4], and F. J. Rybicki[1]

[1]Radiology, Brigham And Womens Hospital & Harvard Medical School, Boston, MA, United States, [2]Radiology, Childrens Hospital & Harvard Medical School, Boston, MA, United States, [3]Radilogy, Brigham And Womens Hospital & Harvard Medical School, Boston, MA, United States, [4]Electrical & Computer Engineering, Northeastern University, Boston, MA, United States

Introduction: For certain applications such as fast imaging and multi-dimensional RF excitation it is advantageous to traverse k-space non-uniformly. However, this introduces the need to compensate the k-space samples (acquired in the context of image reconstruction, or, designed in that of low flip angle RF excitation) for trajectory density variations [1]. Methods that have been proposed for density compensation [2-4] range from analytic solutions applicable to only certain trajectories, to heuristic methods relying e.g., on geometrical arguments such as k-space area, to numerical approaches that attempt to optimize the achieved point spread function, typically using some *ad hoc* criterion. It is shown that the need for density compensation arises when each Fourier basis (F.b.) function corresponding to some k-space location visited by the trajectory is not orthogonal to the F.b. functions corresponding to other k-space locations. When this is the case, the coefficient that effectively becomes associated with each F.b. function is no longer that corresponding to the moment of traversal of the particular k-space location, but rather, it is a linear combination of partial contributions from coefficients corresponding to other locations in the trajectory when the respective F.b. functions have some component parallel to the F.b. function corresponding to the coefficient of interest. An exact linear system can be analytically formed (for typical FOV geometries), and can even be solved non-iteratively for relatively short trajectories (e.g., <20 ms), such as those used in RF excitation. The structure of the system also provides valuable insight into aspects of trajectory design, such as maximum amount of information that can be expressed, and when it may be possible to incorporate e.g., additional constraints in the context of RF excitation.

Theory: Let c_j be the Fourier transform coefficient (to be applied for RF excitation, or, acquired during imaging) corresponding to the k-space location traversed at time $t = j\Delta t$. That is, c_j is the coefficient corresponding to the F.b. function $f_j(\vec{r}) = \exp\{-i2\pi\vec{k}_j \cdot \vec{r}\}$, where $\vec{k}_j$ denotes the k-space location traversed at time t, and $\vec{r}$ denotes spatial location. When the F.b. function $f_j(\vec{r})$ is not orthogonal to some other F.b. functions corresponding to other locations in the traversed k-space, $f_{j'}(\vec{r})$ for $j \neq j'$, we can analyze each $f_{j'}(\vec{r})$ into two components: one parallel to $f_j(\vec{r})$, and one orthogonal to it. Specifically, we may always express $f_{j'}(\vec{r}) = \alpha_{j',j} f_j(\vec{r}) + g(\vec{r})$, where $\alpha_{j',j} = \int_R \left(f_{j'}(\vec{r})\right)\left(f_j(\vec{r})\right)^* d\vec{r}$, and where $\int_R \left(g(\vec{r})\right)\left(f_j(\vec{r})\right)^* d\vec{r} = 0$, where * denotes complex conjugation. Accordingly, when the coefficients c_j are directly Fourier transformed to reconstruct the imaged sample, or, when the low flip angle RF excitation is applied, the effective coefficient that becomes associated with the F.b. function $f_j(\vec{r})$ is not only the desired c_j, but rather it contains contributions from all the $c_{j'}$ corresponding to all F.b. functions with $\alpha_{j',j} \neq 0$. Summing the relevant contributions, a new composite coefficient $b_j = \sum_{j'} c_{j'} \alpha_{j',j}$ [Eq. [1]] emerges that intrinsically becomes associated with the F.b. function $f_j(\vec{r})$ under consideration; density compensation is then the process by which the applied coefficients c_j are chosen so that the effective coefficients b_j that emerge via the linear combination of Eq. [1] are precisely those desired (i.e., those measured during signal acquisition, or those designed for RF excitation). Equation [1] represents a linear system of dimensions $n \times n$, when $j = 1,\ldots n$ (i.e., the trajectory is composed of *n* elements). The integrals of Eq. [1] depend on the region *R* wherein we desire to express information content. When *R* is a square FOV, the integral can be evaluated as $\alpha_{j',j} = \mathrm{sinc}\left\{\pi\left(\vec{k}_j - \vec{k}_{j'}\right)\right\}$, whereas when the region is a disk (as e.g., for spiral trajectories) it can be evaluated as $\alpha_{j',j} = \left(2\left|\left(\vec{k}_j - \vec{k}_{j'}\right)\right|\right)^{-1} J_1\left(\pi\left|\left(\vec{k}_j - \vec{k}_{j'}\right)\right|\right)$, where $J_1(\cdot)$ is the Bessel function of the first kind.

Methods: Experiments were performed on a 1.5T MR scanner (GE Medical Systems, Milwaukee, WI) equipped with 4G/cm, 15 G/cm/ms gradients, and using the body coil for RF receive/transmit (max B_1=250 mG). Three reversed spiral trajectories were designed to maximize slew rate for a 17 cm design FOV: a 5.6 ms 12-loop linearly decreasing density, and a 6 ms 13-loop and a 19 ms 28-loop fixed density trajectories. RF excitations were designed to excite, at 30 degree flip, (a) a square of side 6.2 cm (designed for the 5.6 and 6 ms trajectories), and (b) an arbitrary profile covering a large portion of the FOV (designed for the 19 ms trajectory). The RF excitations were compensated using the heuristic Voronoi tessellation method [3], the iterative Same-Image method [4], and, the proposed method. For the proposed method, the linear system was extended with a simple first order finite difference. This provided regularization so that the system could be solved trivially by Cholesky factorization (0.5 sec computation time), and, produced smooth solutions that are desired for RF excitation fidelity. The 90-degree excitation of a standard spin echo sequence was replaced by the 2D RF pulses so that the 2D profiles were excited orthogonally to the refocusing pulse, which thus refocused only a slice of the profile. This sequence was used to image a spherical phantom occupying a 34 cm FOV so that the first aliasing sidelobe of the excitations could be imaged. Imaging parameters were 7.81 kHz BW, 300 ms TR, 9 ms TE, 20 mm slice thickness, 256 matrix size, 6 NEX, and SNR of ~100. Normalized root-mean-square errors (NRMSEs) were computed for experimental results, as well as for Bloch equation simulations of the excitations.

Results: The figure summarizes results obtained with the 6 ms trajectory and square profile. NRMSEs were lower for the proposed "Cross-Correlations" method. Similar results were obtained for the other trajectories and profiles. The final column shows the case where the designed RF field was compensated for a square FOV. In general this is not possible for spirals since the first aliasing sidelobe is a ring. This was verified by results (not shown) obtained for the solution obtained by the proposed method for a square FOV compensation, and the arbitrary excitation profile that covered the majority of the disk FOV. However, when it is known that the extent of the desired excitation profile is sufficiently smaller than the design FOV of the spiral, it can be seen that it is possible to obtain a small excitation error even for a square FOV. Note the NRMSEs of the last column were computed within the entire plotted square, whereas for the other methods only the difference within a disk was used.

Acknowledgements: Whitaker Foundation, NIH K23-EB00882.

References: [1] Jackson et al. *IEEE TMI*. 1991;10:473-8. [2] Hoge et al. *MRM*. 1997;38:117-28. [3] Rasche et al. *IEEE TMI* 1999;18:385-92. [4] Qian et al. *MRM*. 2002;48:306-11.

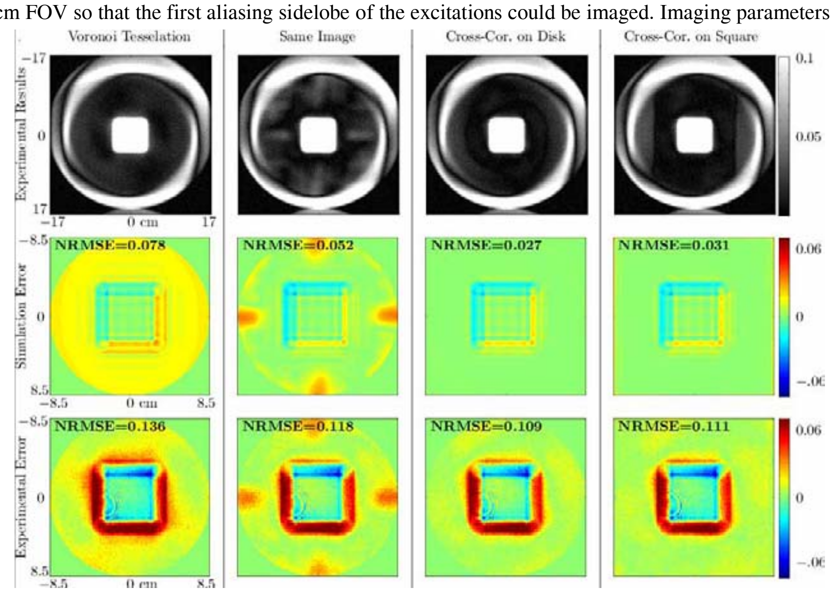

Figure 1. Top row: experimentally measured excitation profiles for a 6 ms reversed spiral trajectory and RF field designed to produce a square of 6.2 cm and compensated using different density compensation methods. Middle and bottom rows: difference between simulated and experimental excitation profiles and desired profile designed.