Multidimensional Arbitrary-Flip-Angle Parallel Transmit Pulse Design Using an Optimal Control Approach

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INTRODUCTION

The majority of the design methods for parallel transmit radiofrequency (RF) pulses so far are based on small-tip-angle (STA) approximation of the Bloch equation [1-3]. These methods can design only excitation pulses with a small flip angle (≤ 90°). We have proposed a noniterative method to design large-tip-angle (LTA) parallel transmit pulses (> 90°), based on an extension of the single channel linear class LTA (LCLTA) theory [4]. However, both STA and LCLTA are linear approximations of the nonlinear Bloch equation. As a result, imperfections due to higher order terms (when the corresponding assumptions are violated) can appear in the final magnetization profiles. Although numerical methods [5] have been proposed to improve the linear solution in 1D, single channel case, the feasibility to extend these methods to multidimensional, parallel transmit cases has not been studied yet. In this paper, we formulate the multidimensional, parallel transmit RF pulse design as a multidimensional, multi-controller optimal control problem, which can be numerically solved by a first-order gradient algorithm. Both Bloch simulation and parallel transmit experiments have been carried out to verify the effectiveness of the pulses designed by the proposed method for reduced field-of-view (FOV) imaging.

PROPOSED METHOD

We denote $b_1^{(l)}(t)$ as the RF pulse to be designed and $s_l(\mathbf{r})$ as the transmit sensitivity for the lth coil, $l = 1,2,\ldots,L$. The effective B_1 for parallel transmit can be written as:

$$B_1(\mathbf{r},t)=\sum_{l=1}^{L}s_l(\mathbf{r})b_1^{(l)}(t)=\sum_{l=1}^{L}[s_l^{(\mathrm{R})}(\mathbf{r})u_l(t)-s_l^{(\mathrm{I})}(\mathbf{r})v_l(t)]+i\sum_{l=1}^{L}[s_l^{(\mathrm{I})}(\mathbf{r})u_l(t)+s_l^{(\mathrm{R})}(\mathbf{r})v_l(t)]\,, \quad (1)$$

where $s_l^{(\mathrm{R})}(\mathbf{r}) = \mathrm{Re}\{s_l(\mathbf{r})\}$, $s_l^{(\mathrm{I})}(\mathbf{r}) = \mathrm{Im}\{s_l(\mathbf{r})\}$, $u_l(t) = \mathrm{Re}\{b_1^{(l)}(t)\}$, and $v_l(t) = \mathrm{Im}\{b_1^{(l)}(t)\}$. Using Eq. (1), the Bloch equation (without relaxation) under $B_1(\mathbf{r},t)$ and gradient $\mathbf{G}(t)$ can be written as:

$$\dot{\mathbf{M}}(t)=[\mathbf{A}(t)+\sum_{l=1}^{L}\mathbf{B}^{(l)}u_l(t)+\sum_{l=1}^{L}\mathbf{C}^{(l)}v_l(t)]\mathbf{M}(t)\,, \quad (2)$$

where $\mathbf{M}(t)$ is a vector that contains magnetization components of all three axes (x, y, and z) at all spatial locations, $\mathbf{A}(t)$ is a matrix that contains the gradient terms, $\mathbf{B}^{(l)}$ and $\mathbf{C}^{(l)}$ are two constant matrices containing the sensitivity terms. The multidimensional, parallel transmit pulse design problem is formulated as to find $u_l(t)$ and $v_l(t)$ to minimize the final magnetization error regularized by RF power:

$$\text{Minimize } J[u_1(t),\cdots,u_L(t),v_1(t),\cdots,v_L(t)]=\frac{1}{2}[\mathbf{M}(T)-\mathbf{D}]^{\mathrm{T}}\mathbf{W}[\mathbf{M}(T)-\mathbf{D}]+\frac{1}{2}\alpha[\sum_{l=1}^{L}u_l^2(t)+\sum_{l=1}^{L}v_l^2(t)]\,, \quad (3)$$

where $\mathbf{M}(T)$ is the final magnetization vector that satisfies Eq. (2) (T is pulse duration), $\mathbf{D}$ is the desired magnetization vector, $\mathbf{W}$ is a diagonal matrix containing weights for different spatial locations to improve the accuracy in regions-of-interest, and α is a parameter to balance the two terms in J. Equation (3) is a multidimensional, multi-controller optimal control problem. Similarly to the single channel case [5], further derivation shows Eq. (3) leads to a two-point boundary-value problem, which does not have a closed-form solution. We solve Eq. (3) using a first-order gradient algorithm [6], with the initial guess being an STA or LCLTA pulse.

RESULTS

Bloch Simulation. We applied the proposed method to optimize an eight-channel 2D 180° inversion pulse to invert an infinite cylinder (diameter = 8 cm) in an FOV of 16 × 16 cm^2. The initial magnetization vector pointed to $+z$ axis. The transmit sensitivities were created by an FDTD software to simulate an 8-channel head transmit array at 7 T main field strength. We adopted a 10-turn inherently refocused spiral trajectory [4] to cover the k-space (Fig 1a). We used LCLTA method [4] to design the initial pulse and then applied the proposed method to obtain an optimized pulse. Imaginary components of one channel of the pulses are shown in Fig. 1b (T = 5 msec). As shown in Figs. 1c-d, although M_x component stays close to zero at most spatial locations, some initial magnetization indeed gets flipped to the x axis, resulting in large errors in the final M_x profile; M_z is inverted inside the cylinder, but some distortion due to the nonlinearity of Bloch equation is noticeable. These errors were well corrected by the proposed method, as shown in Figs. 1e-f: the M_x component stays closer to zero than that from the LCLTA; M_z component has much higher uniformity inside the cylindrical region than that from the LCLTA solution. This result suggests that the proposed method can help improve the uniformity inside the selected region-of-interest.

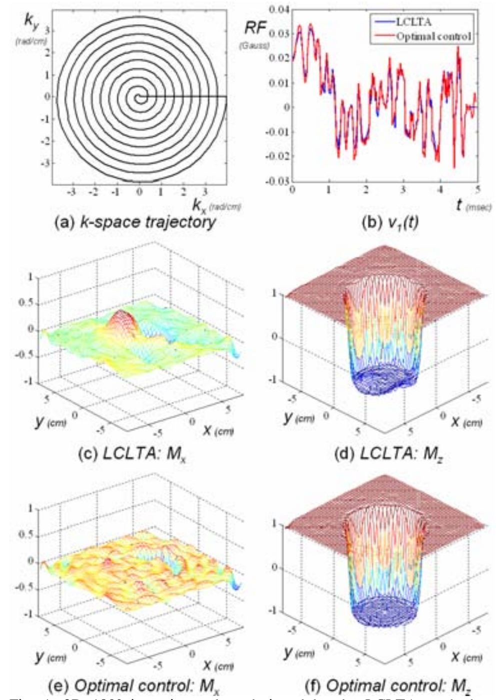

Fig. 1. 2D 180° inversion pulses designed by the LCLTA method and optimized by the proposed method for reduced field-of-view imaging: (a) K-space trajectory. (b) Imaginary components of one channel of the pulses. (c-d) M_x and M_z components produced by the LCLTA pulse. (e-f) M_x and M_z components produced by the optimized pulse.

Experiments. A spin-echo experiment on a 1.5 T GE Signa scanner with parallel transmit capability [7] was carried out to compare non-optimized and optimized excitation and refocusing pulses. The target profile was an off-centered cylinder (diameter = 6 cm). FOV = 30 × 30 cm^2, TE = 15 msec, TR = 100 msec. An eight-channel transmit-only head-coil array [7] was used to transmit RF pulses and a single channel quadrature body coil was used to receive MR signal. The method in [7] was used for B_1 mapping. The 90° excitation pulse using a twofold accelerated inward spiral trajectory was designed using the STA method [3]. The 180° refocusing pulse using a twofold accelerated inherently refocused spiral trajectory [4] was designed using the LCLTA method (detailed in another abstract submitted to this conference). The resulting spin-echo image is shown in Fig. 2a. Although good spatial selectivity is achieved, noticeable artifacts due to Bloch equation nonlinearity still occur. As shown in Fig. 2b, the image produced by the optimized pulses (optimized 90° + optimized 180°) shows a very clean "background" outside of the target region. This result implies that the proposed method can help reduce background "ripples" in reduced FOV imaging.

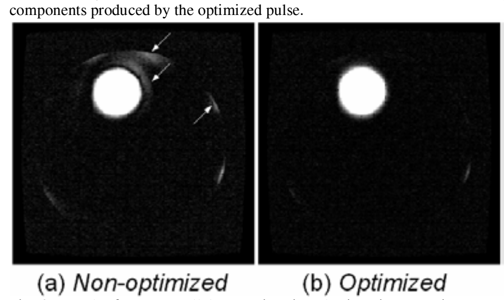

Fig. 2. Results from a parallel transmit spin-echo imaging experiment. (a) Image by the non-optimized pulses (STA 90° + LCLTA 180°). Some artifacts due to Bloch nonlinearity occur as marked by the arrows. (b) Image by the optimized pulses, which has a very clean "background".

CONCLUSION

Directly based on the Bloch equation, we formulate the multidimensional, parallel transmit RF pulse design as a multidimensional, multi-controller optimal control problem, which can be numerically solved by a first-order gradient algorithm. Bloch-simulation and experimental results show that the optimized pulses (with arbitrary flip angles) can generally lead to significantly better magnetization profiles than the non-optimized pulses.

REFERENCES

[1] Katscher et al., *MRM*, vol. 49, pp. 144-150, 2003.
[2] Y. Zhu, *MRM*, vol. 51, pp. 775-784, 2004.
[3] W. A. Grissom et al., *Proc. ISMRM*, pp. 19, 2005.
[4] J. Pauly et al., *JMR*, vol. 82, pp.647-654, 1989.
[5] Conolly et al., *IEEE-TMI*, vol. MI-5, pp. 106-115, 1986
[6] Bryson et al., *Applied Optimal Control*, Hemisphere, 1975.
[7] Y. Zhu et al., *Proc. ISMRM*, pp. 14, 2005.