DESIGNING FAST 3-D RF EXCITATIONS BY OPTIMIZING THE NUMBER, PLACEMENT AND WEIGHTING OF SPOKES IN *K*-SPACE VIA A SPARSITY-ENFORCEMENT ALGORITHM

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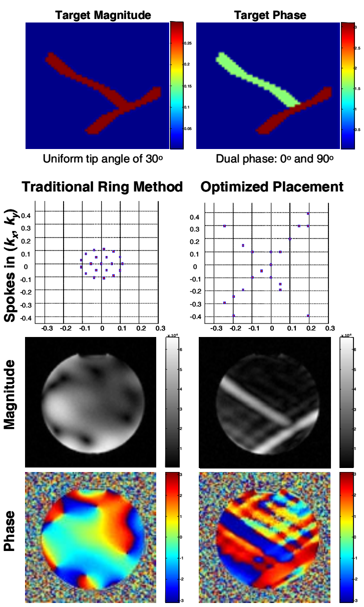

INTRODUCTION: Playing sinc-like RFs in the presence of slice-selective gradient trajectories is useful for exciting a thin slice in z and is analogous to placing sinc-like "spokes" along k_z in excitation k-space. Recently, the use of multiple complex-weighted spokes at different locations in the (k_x, k_y) plane has led to RF pulses that mitigate B_1-inhomogeneity in single-coil excitation systems [1] and reduce excitation time (TE) on multi-channel systems [2]. While placing a tall spoke in k-space does indeed significantly increase pulse duration and minimum TE, it is a necessary tradeoff to ensure a sharp slice profile with low sidelobes. Because of this high temporal cost per spoke, an *ideal* thin-slice design would be one that used very few spokes while achieving a user-specified excitation in the (x,y) plane with high-fidelity. To achieve this, we propose an algorithm that optimizes the number, placement and weighting of spokes, based on sparse approximation theory. First we show the theory encompasses RF design for multi-channel excitation systems, with single-channel systems as a base case. We then show the method generates fast, high-fidelity slice-selective pulses, achieving near-optimal tradeoff of TE & excitation fidelity. Experiments conducted in a phantom on a 3T Siemens Magnetom TRIO with an 8-channel parallel TX array show the algorithm's advantages over traditional (k_x, k_y) spoke placement patterns.

METHODS AND RESULTS: **Sparse approximation (SA)**. The goal of SA is to find a vector or matrix of unknowns *with a small number of nonzero elements* such that a system of equations approximately holds, e.g., $\mathbf{m} = \sum_{p=1}^{P} \mathbf{F}_p \boldsymbol{\varphi}_p + \mathbf{n}$, where $\mathbf{m}, \mathbf{n} \in C^M$, $\mathbf{F}_p \in C^{M \times N}$, $\boldsymbol{\varphi}_p \in C^N$, and $N > M$. This problem is ill-posed because there are infinitely many choices of $\boldsymbol{\varphi}_p$ vectors that solve it. But consider enforcing *sparsity* on the $\boldsymbol{\varphi}_p$, requiring the l_1-norm of each to be small, which is similar to requiring many elements of each $\boldsymbol{\varphi}_p$ to equal zero [3]. Suppose we further constrain the $\boldsymbol{\varphi}_p$, requiring them to be *simultaneously sparse*: each of the $\boldsymbol{\varphi}_p$ must have nonzeros occurring at a similar set of indices. With such requirements, the problem is no longer ill-posed. Letting $\boldsymbol{\Phi} = [\boldsymbol{\varphi}_1, \ldots, \boldsymbol{\varphi}_P]$, a program that finds a simultaneously sparse set of $\boldsymbol{\varphi}_P$ and approximately yields $\mathbf{m}$ is as follows: $\min_{\boldsymbol{\Phi}} \; (1-\lambda)\left\| \mathbf{m} - \left(\sum_{p=1}^{P} \mathbf{F}_p \boldsymbol{\varphi}_p \right) \right\|_2 + \lambda \left\| \boldsymbol{\Phi} \right\|_S$, where the second term, $\|\boldsymbol{\Phi}\|_S$, is the l_1-norm of the l_2-norms of the rows of $\boldsymbol{\Phi}$, a *simultaneous sparsity norm* that penalizes (rewards) the program when the columns of $\boldsymbol{\Phi}$ have dissimilar (similar) sparsity profiles. The first term keeps the residual error down. As λ increases from 0 to 1, sparser solutions are generated while the residual error increases, i.e., λ trades off sparsity with residual error. Because the objective function is convex, there exists an optimal solution $\boldsymbol{\Phi}^*$ that attains the global minimum. $\boldsymbol{\Phi}^*$ may be computed via a Second-Order Cone program. Refer to [3,4] for more details.

Proposed algorithm. Our goal is to excite a thin, sharp slice that approximately equals a user-specified target excitation $m(x,y)$ at z=z_0 and zero at $z \neq z_0$. To accomplish this, we must decide on a number of spokes to use, their locations in (k_x, k_y), and weights for each. Using spokes in k_z will let us obtain a thin slice in z. But achieving the in-slice target $m(x,y)$ is more complicated: ideally, many weighted spokes would be placed in (k_x, k_y) such that $m(x,y)$ was almost exactly achieved, but this would require many spokes and result in a long TE. To keep TE short, we must use a *small* number of spokes, but with few spokes, achieving $m(x,y)$ becomes difficult. Let us define $\mathbf{m}$ to be vector of spatial samples of the $m(x,y,z_o)$ excitation in some region of interest (ROI).

Analogy between spokes and Diracs in 2-D Fourier domain. Placing a spoke in (k_x, k_y) with some arbitrary complex weight φ is analogous to placing a weighted-Dirac delta, $\varphi\delta(k_x,k_y)$ in the 2-D Fourier domain. Since spokes are expensive in terms of pulse duration, each δ is also expensive. Using this analogy, our goal is now: *using a* small *number of complex-weighted δ's in 2-D Fourier space, ensure that their 2-D Fourier transform is close to m(x,y) at all points in the region of interest.*

Base case (*P* = 1) formulation. Assume a finite grid of discrete points exists in (k_x, k_y). Each point is a Dirac delta that produces a complex exponential in the spatial domain. An arbitrary choice of complex weights at different points on the grid results in a spatial domain excitation related to the weighted grid by a Fourier transform. Arranging the complex weights of this grid into the vector $\boldsymbol{\varphi}_1$, the following holds: $\mathbf{r} = \mathbf{D}_1\mathbf{A}\boldsymbol{\varphi}_1 = \mathbf{F}_1\boldsymbol{\varphi}_1$, where $\mathbf{r}$ is a vector of spatial samples of the *resulting* (x,y,z_o) excitation in the ROI, $\mathbf{D}_1$ is a diagonal matrix of samples of the coil sensitivity pattern in the ROI, and $\mathbf{A}_{m,n}=\exp(j2\pi k_x[n]x[m]+k_y[n]y[m])$ [5]. If the n-th element of $\boldsymbol{\varphi}_1$ is nonzero, this corresponds to a spoke at $(k_x(n), k_y(n))$. Thus, a *sparse* $\boldsymbol{\varphi}_1$ that results in an $\mathbf{r}$ close to $\mathbf{m}$ is ideal: it implies a short, high-fidelity excitation.

Extension to parallel systems (*P* > 1). For parallel systems, the formulation extends as follows: $\mathbf{r} = \mathbf{D}_1\mathbf{A}\boldsymbol{\varphi}_1 + \cdots + \mathbf{D}_P\mathbf{A}\boldsymbol{\varphi}_P = \mathbf{F}_1\boldsymbol{\varphi}_1 + \cdots + \mathbf{F}_P\boldsymbol{\varphi}_P$, with the constraint that the $\boldsymbol{\varphi}_p$ must be simultaneously sparse, which physically means that the RF pulses along each of the P coils (the $b_{1,p}(t)$ waveforms) must each play along the *same* k-space trajectory. This constraint arises because the system's set of gradients determines a *unique* k-space trajectory $k(t)$. If the $\boldsymbol{\varphi}_p$ were not simultaneously sparse, it would imply the RFs are concurrently played along P *different* k-space trajectories, which is not possible.

Step I: determine spoke locations. Using as few points on the frequency grid as possible, we want to attain the user-specified $\mathbf{m}$ within the thin-slice, i.e., we want to find a *simultaneously sparse* $\boldsymbol{\Phi}$ matrix such that the residual error term $\left\|\mathbf{m} - \mathbf{r}\right\|_2 = \left\| \mathbf{m} - \left(\sum_{p=1}^{P} \mathbf{F}_p \boldsymbol{\varphi}_p \right) \right\|_2$ is small. Finding this $\boldsymbol{\Phi}$ is accomplished by fixing λ and solving the optimization program above. With the proper choice of λ, a simultaneously sparse, globally optimal $\boldsymbol{\Phi}$ matrix is found that keeps the residual error down.

Step II: keep *T* spokes and determine *PT* weights. Since each row of $\boldsymbol{\Phi}$ that contains nonzeros corresponds to a spoke that must be traversed in k-space, we zero out all but T rows of $\boldsymbol{\Phi}$, keeping those with the largest l_2-energy. Thus, T is a control parameter explicitly trading off the number of spokes, and hence TE, with excitation fidelity. Since all but T rows of $\boldsymbol{\Phi}$ equal zero, the affine system of equations is now reduced to $\mathbf{r} = \sum_{p=1}^{P} \mathbf{F}_{T,p} \boldsymbol{\varphi}_{T,p}$, where each $\mathbf{F}_{T,p}$ is a truncated matrix whose T columns correspond to the T columns of $\mathbf{F}_p$ that remain after discarding all but T rows of $\boldsymbol{\Phi}$. Now, $\boldsymbol{\Phi}_T$ is recalculated to attain an excitation closer to $\mathbf{m}$, i.e., the weights at each of the T spoke locations are retuned for each of the P coils in a least-squares sense via the pseudo-inverse of $[\mathrm{F}_{T,1}, \ldots, \mathrm{F}_{T,P}]$.

Step III: generate an RF pulse set. At this point, T spokes have been placed in k-space at points on the (k_x, k_y) grid implicitly defined by the $\boldsymbol{\varphi}_p$. Further, P weights have been determined for each spoke (one per excitation coil). With this information, a set of RF pulses may be designed using the method in [2].

Results. The proposed method was compared to a non-optimized spoke placement on a grid within a fixed radius. Experiments were conducted in a phantom on a 3T Siemens Magnetom TRIO equipped with an 8-channel TX array. Each method used 20 spokes and both attempted to excite the dual-phase bifurcation target shown above. In the upper right, the spoke placement in (k_x, k_y) used by the traditional method, the optimized spoke placement calculated by the proposed technique, and each method's resulting excitations are depicted. The proposed algorithm's near-optimal spoke placement led to an excitation closely resembling the target. For each design, TE was ~10 ms and slice thickness was 1 cm. Because both methods used the same sinc-like spokes in k_z, they exhibited equivalent slice-selectivity performance.

CONCLUSION: An algorithm that optimized the number, placement and weighting of spokes in k-space was presented and extended from single- to multi-channel TX systems. An experiment using an 8-channel TX array on a Siemens Magnetom TRIO showed that the proposed algorithm outperformed a non-optimized technique.

ACKNOWLEDGEMENTS & REFERENCES: Siemens Medical Solns., NIH P41RR14075, US DoD NDSEG Fellowship, R.J. Shillman Career Dev. Award. **[1]** Saekho et al. Fast-k_z 3-D Tailored RF Pulse for Reduc B_1 Inhomog. *MRM*. 2006;55(4):719-724. **[2]** Setsompop et al. Parallel RF Trans w/ 8 Chan at 3T. *MRM*. 2006;56(5). **[3]** Chen et al. Atomic Decomp by Basis Pursuit. *SIAM Rev*, 2001;43(1):129–159. **[4]** Malioutov et al. Sparse Signal Recon Perspec for Source Localiz with Sensor Arrays. *IEEE Trans Sig Proc.*, 2005;53(8). **[5]** Grissom et al. Spatial Domain Method for the Design of RF Pulses in Multicoil Parallel Excit. *MRM*. 2006;56(3):620-629.